ERRATA

Statement and proof of Lemma 15 p. 1221 are changed as follows.

LEMMA 15. *Let G be a graph on a set Q of vertices. Let $i \in Q$ and $T \subset Q$. If G admits a positive k-approximate eigenvector $\mathbf{w}$, there is a graph G' and a set of vertices I' of G' such that the following hold*

1. *G' admits the k-approximate eigenvector $\mathbf{w}'$ with all components equal to 1.*
2. *the triple $(G, i, \mathbf{w})$ is equivalent to the triple $(G', I', \mathbf{w}')$;*
3. *If $w_p = 1$ for all $p \in T$, there is a set of vertices T' of G' such that the triple (G, i, T) is equivalent to the triple (G', I', T'). Moreover, if T is the set of leaves of G, there is subgraph G'' of G' with T' as set of leaves, and a subset I'' of I' such that the triple (G, i, T) is equivalent to the triple (G'', I'', T'), and G'' has a k-approximate eigenvector with all components equal to 1.*

Proof. Let M be the adjacency matrix of G. Let Q' be the set of pairs (q, j) with $q \in Q$ and $1 \leq j \leq w_q$. For each $p \in Q$, let us consider the set of triples $\{(q, j, l) \mid q \in Q, 1 \leq j \leq w_q, 1 \leq l \leq M_{pq}\}$. Its cardinality is $\sum_{q \in Q} M_{pq} w_q$. Since for each $p \in Q$, we have

$$\sum_{q \in Q} M_{pq} w_q \leq k w_p,$$

it is possible to partition this set in w_p sets $X_{(p,1)}, X_{(p,2)}, \ldots, X_{(p,w_p)}$ of at most k elements. We now define the $Q' \times Q'$ square matrix M' by setting, for $p, q \in Q$, $1 \leq i \leq w_p$ and $1 \leq j \leq w_q$, $M'_{(p,i)(q,j)}$ as the number of triples in $X_{(p,i)}$ whose first two components are (q, j). Let U be the $Q \times Q'$ matrix defined by $U_{q(q,j)} = 1$, for any $1 \leq j \leq w_q$, the other components being zero. By construction, we get

$$UM' = MU.$$

Indeed, the coefficient of index $p, (q, j)$, where $1 \leq j \leq w_q$, of MU is

$$\sum_{r \in Q} M_{pr} U_{r(q,j)} = M_{pq}.$$

And the coefficient of index $p, (q, j)$ of UM' is

$$\sum_{(r,i) \in Q'} U_{p(r,i)} M'_{(r,i)(q,j)} = \sum_{1 \leq i \leq w_p} M'_{(p,i)(q,j)} = M_{pq}.$$

Let $\mathbf{i}$ the Q-row characteristic vector of i. We define the Q'-row vector $\mathbf{i}'$ by $\mathbf{i}' = \mathbf{i}U$. It is the characteristic vector of $I' = \{(i, j) \in Q' \mid 1 \leq j \leq w_i\}$. We denote by G' the graph whose set of vertices is Q' and whose adjacency matrix is M'. Let $\mathbf{w}'$ be the Q'-column vector whose all components are 1. It is straightforward that $\mathbf{w} = U\mathbf{w}'$.

Since $UM' = MU$, $\mathbf{i}' = \mathbf{i}U$ and $\mathbf{w} = U\mathbf{w}'$, G is a multiset extension of G'. Then, according to Proposition 10, the triple $(G, i, \mathbf{w})$ is equivalent to the triple $(G', I', \mathbf{w}')$. Since the sum of each row of M' is less than or equal to k, $\mathbf{w}'$ is a right k-approximate eigenvector of M'.

The third point of the Lemma is obtained as follows. Let $\mathbf{t}$ the Q-row characteristic vector of the set T. Since $w_p = 1$ for all $p \in T$, $\mathbf{t} \leq \mathbf{w}$. Let us define the

Q'-column vector $\mathbf{t}'$ by $t'_{(q,j)} = 1$ if and only if $1 \le j \le t_q$ and $t'_{(q,j)} = 0$ otherwise. Then, for $p \in Q$,

$$(U\mathbf{t}')_p = \sum_{(q,j)\in Q'} U_{p(q,j)} t'_{(q,j)} = \sum_{1\le j\le w_p} t'_{(p,j)} = t_p.$$

We get $\mathbf{t} = U\mathbf{t}'$. The vector $\mathbf{t}'$ is the characteristic column vector of the set $T' = \{(p,1) \in Q' \mid p \in T\}$. It follows that (G,i,T) is equivalent to the triple (G',I',T'). If T is the set of vertices which have no outgoing edges, then, for any $q \in Q$, any $1 \le j \le w_q$ and any $p \in T$, $M_{pq} = 0$, and $M'_{(p,1)(q,j)} = 0$. Thus the vertices of T' have no outgoing edges. One can finally remove from G' and I' the leaves which do not belong to T', until obtaining a graph G'' whose set of leaves is T', and such that (G'',I'',T') is equivalent to (G,i,T). If Q'' is the set of vertices of G'', the Q''-column vector with all components equal to 1 is a k-approximate eigenvector of G''. $\square$