THE TOP TEN THEOREMS ON PARTIALLY ORDERED SETS

WILLIAM T. TROTTER

Abstract

A few years ago, I gave an impromptu lecture with the title "The Top Five Theorems on Posets." It was a light hearted affair, but in retrospect it got the audience to thinking about whether they agreed or disagreed with my choices, and it certainly caused me to think more deeply about the most significant threads for research on partially ordered sets. So in this talk, I propose to revisit this theme, expanding from five to ten to give me a little more room. Some choices are obvious. You have to include Dilworth's chain partitioning theorem and Sperner's lemma giving the width of the subset lattice. Here are three more: (1) Schnyder's theorem testing planarity for graphs in terms of the dimension of the incidence poset; (2) Canfield's disproof of Rota's cross-cut conjecture for the partition lattice; and (3) Brightwell, Felsner and Trotter's improvement to the Kahn/Saks theorem on balancing pairs. So what are the other five? Hints are given in the following reading list—but no doubt there will be a few surprises.

References

[1] R. Ahlswede and D. E. Daykin, An inequality for the weights of two families of sets, their unions and intersections, *Z. Wahrscheinlichkeitstheorie und Verw. Gebiete* **43** (1978), 183–185.

[2] N. Alon and E. R. Scheinerman, Degrees of freedom versus dimension for containment orders, *Order* **5** (1988), 11–16.

[3] G. R. Brightwell, S. Felsner and W. T. Trotter, Balancing pairs and the cross product conjecture, *Order* **12** (1995), 327–349.

[4] S. Felsner, P. C. Fishburn and W. T. Trotter, Finite three dimensional partial orders which are not sphere orders, *Discrete Math.* **201** (1999), 101–132.

[5] P. C. Fishburn, A correlational inequality for linear extensions of a poset, *Order* **1** (1984), 127–137.

[6] W. Schnyder, Planar graphs and poset dimension, *Order* **5** (1989), 323–343.

[7] R. P. Stanley, Two combinatorial applications of the Aleksandrov/Fenchel inequalities, *J. Combin. Theory Ser. A* **31** (1981), 56–65.

E-mail address: `trotter@ASU.edu`

Department of Mathematics, Arizona State University, Tempe, AZ 85287, USA