

THE  
THEORY OF  
DETERMINANTS  
in the  
HISTORICAL ORDER  
OF DEVELOPMENT

VOLUME TWO  
THE PERIOD 1841 TO 1860.

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## PREFACE.

THE story of the production of this volume is quite similar to that of the first. Its bibliographical basis was the three "Lists of Writings on Determinants" formerly referred to (see *History*, I., Introduction). These were followed at irregular intervals by a long series of papers in the *Proceedings of the Royal Society of Edinburgh*, containing historically arranged accounts of the writings in question up to 1860. A certain amount of outside attention having thus been attracted to the subject, neglected writings known to other mathematicians came gradually to be brought to my notice, and of course these were steadily supplemented by assiduous research on my own part. As a consequence, the "Fourth List of Writings," published in 1906,\* was able to show a score or so of titles belonging to the period prior to 1860, and the "Fifth List," published about a fortnight ago,† a still greater number of similarly belated items. Considerable intercalations have thus been made in the Edinburgh series of papers, and much care has been bestowed on a complete revision of the whole. It is confidently hoped that now very little matter of real moment has escaped attention.

T. M.

CAPETOWN, SOUTH AFRICA,  
19th July, 1911.

\* *Quart. Journ. of Math.*, xxxvii. pp. 237-264.

† *Quart. Journ. of Math.*, xlii. pp. 343-378.



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