graphotaxy: a graph classification system

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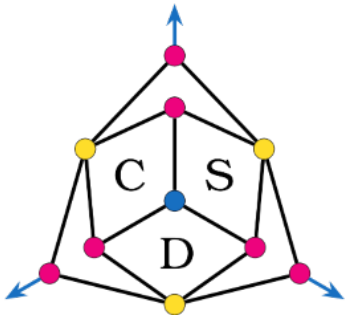

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I focus on two problems:

1. **recognition:** given a graph G and a graph class $\mathcal{C}$, decide whether $G \in \mathcal{C}$;
2. **classification:** given a graph G, determine all classes to which G belongs.

Why?

Additional knowledge on the structure of graphs under consideration is valuable in various areas (see e.g. all previous talks). Most notably:

- **algorithms:** "hard" problems may become "easy" or "easier" on particular graph classes (NPC $\rightarrow$ P, W $\rightarrow$ FPT, ...);

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- **"pure" graph theory:** how does this new class relate to those famous classes (inclusion, separation, ...)?
- **extremal graph theory:** what is / are the graph class(es) that optimise this invariant's value?

Scope

`graphotaxy` only deals with classes that appear in the ISGCI (*Information System on Graph Classes and their Inclusions*).

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This is "an encyclopaedia of graph classes", which features a wealth of information for each class. Most importantly for me:

1. its recognition status (in P, NP-complete, or unknown);
2. its minimal superclasses and its maximal subclasses in ISGCI;

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But let's not give up just yet.

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As of today, 728 recognisers are currently implemented, covering 1069 classes out of 1690 (63.25 % coverage).

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You can also restrict the scope of `graphotaxy` using these two options:

- `--only ids`: classes to which the classification must be restricted;
- `--skip ids`: classes whose recognition should be skipped;

(use ISGCI ids: "bipartite" = gc_69, "perfect" = gc_56, ...)

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What's better than an efficient recognition algorithm?
An algorithm you don't run.

Inclusion relationships

I build the **ISGCI inclusion digraph** $\mathscr{G} = (V, A)$, with:

- $V =$ all graph classes in ISGCI;
- A contains an arc $(\mathcal{C}, \mathcal{D})$ iff $\mathcal{D}$ is a maximal subclass of $\mathcal{C}$.

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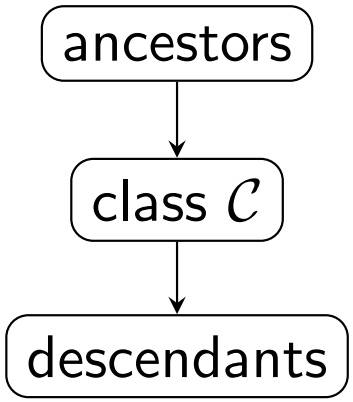

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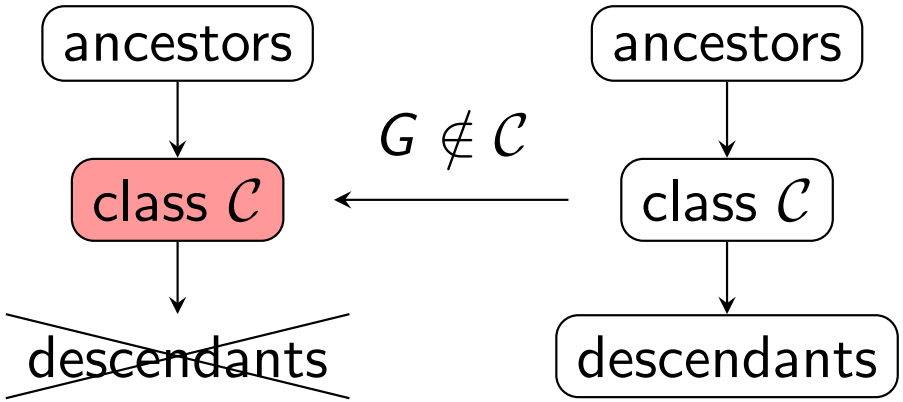

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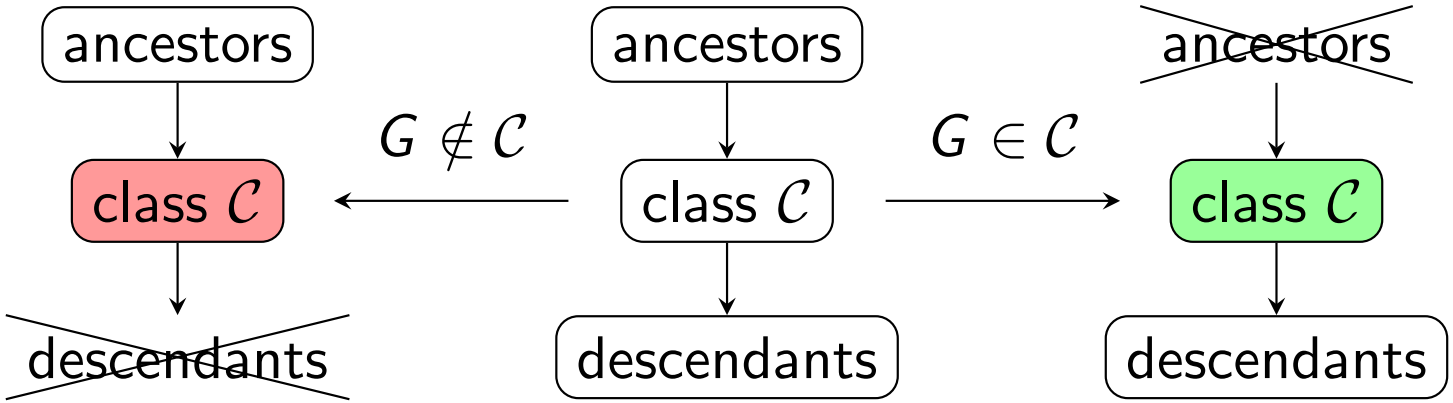

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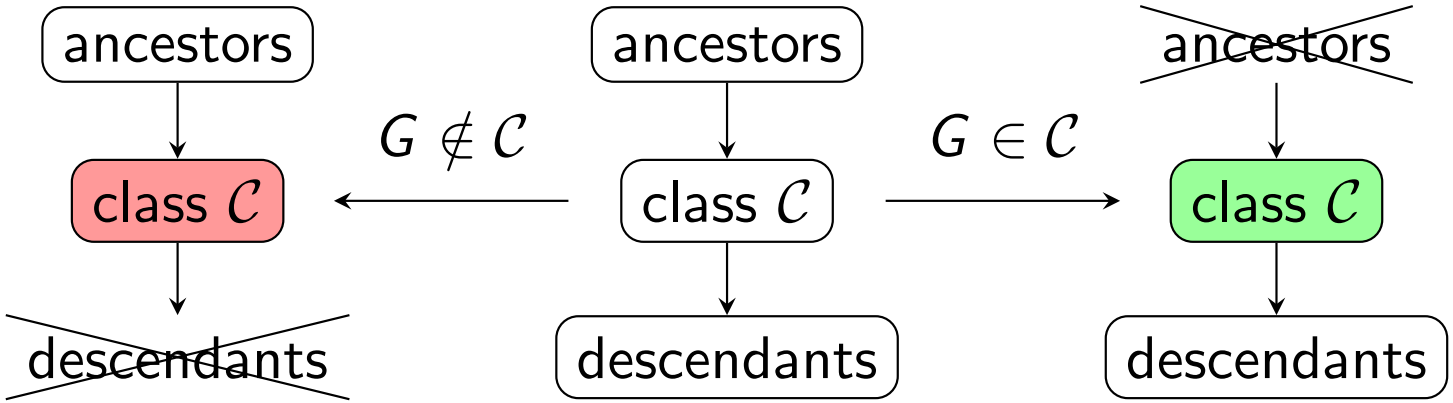

These **propagations** allow me to avoid running many recognition algorithms at all.

Exclusion relationships

Besides inclusion relationships (i.e., "$G \in \mathcal{C} \Rightarrow G \in \mathcal{D}$"), I also take advantage of **exclusion relationships** (i.e., "$G \in \mathcal{C} \Rightarrow G \notin \mathcal{D}$").

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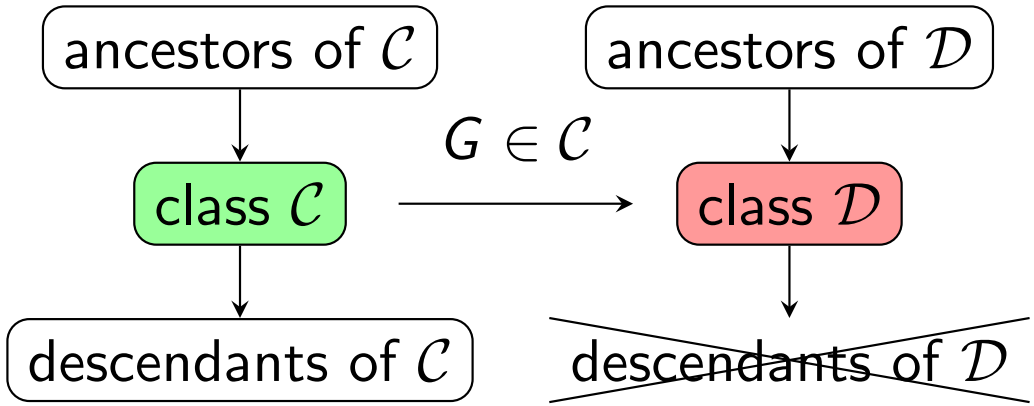
Besides inclusion relationships (i.e., "$G \in \mathcal{C} \Rightarrow G \in \mathcal{D}$"), I also take advantage of **exclusion relationships** (i.e., "$G \in \mathcal{C} \Rightarrow G \notin \mathcal{D}$"). For instance:

- k-regular for $k > 1 \Rightarrow \neg$ tree (trivial);
- unbreakable $\Rightarrow \neg$ P_4-free (less trivial);
- ⋮

ISGCI doesn't provide these, so collecting them takes some time (**please send me yours!**).

Exclusion relationships

The idea is similar to what we've seen regarding inclusion relationships: if $G \in \mathcal{C} \Rightarrow G \notin \mathcal{D}$, then:



And so I can skip even more classes in addition to the earlier propagations.

Forbidden induced subgraphs

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$\Rightarrow$ naïve recognition algorithm: for every H in S, check whether any $|V_H|$-subset of V_G induces a subgraph isomorphic to H;

✓ good for coverage (about 400 such classes);

✗ slow – but let's work on that.

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  3. avoid running other recognisers entirely, by first inspecting their FISC;

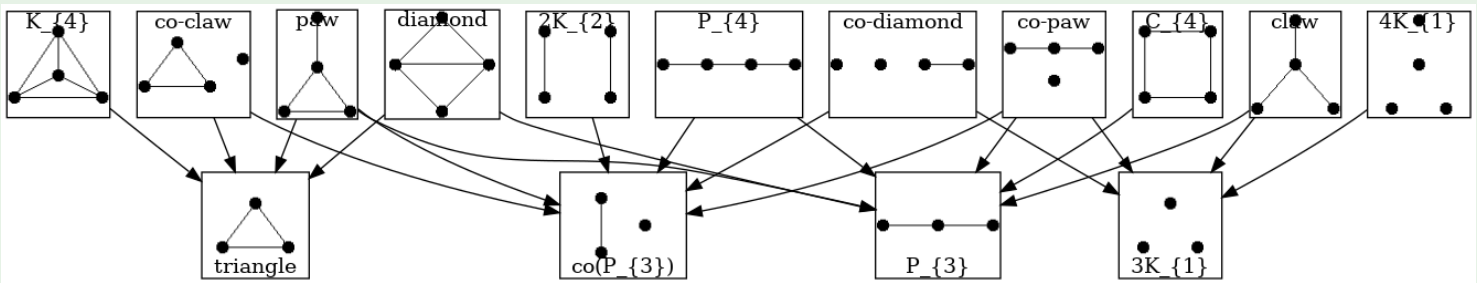
Propagations for patterns

I build a **smallgraph inclusion digraph** $\mathscr{S} = (V, A)$, where:

- $V \supseteq$ all smallgraphs in ISGCI ($|V| = 611$);
- A contains an arc (G, H) iff H is an induced subgraph of G.

Example

The subgraph of $\mathscr{S}$ induced by all smallgraphs of sizes 3 and 4:



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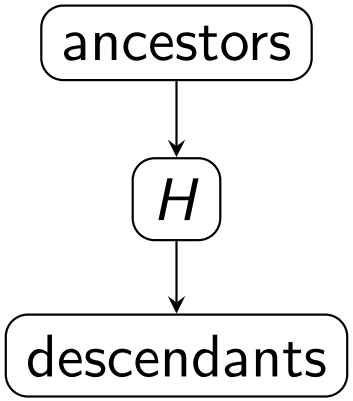

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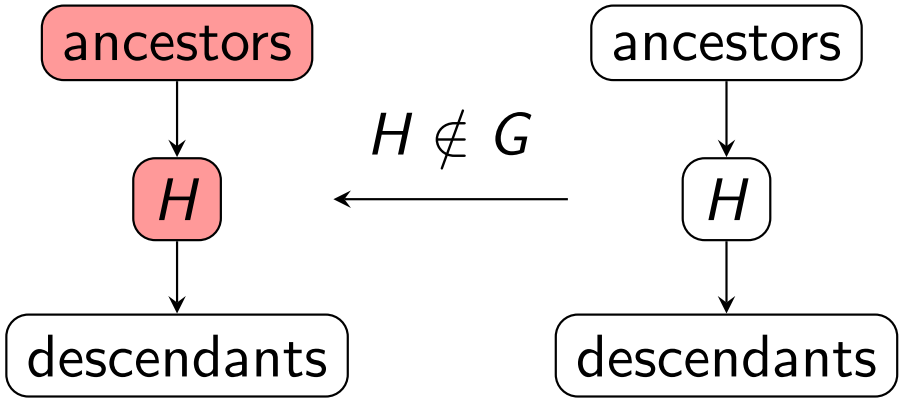

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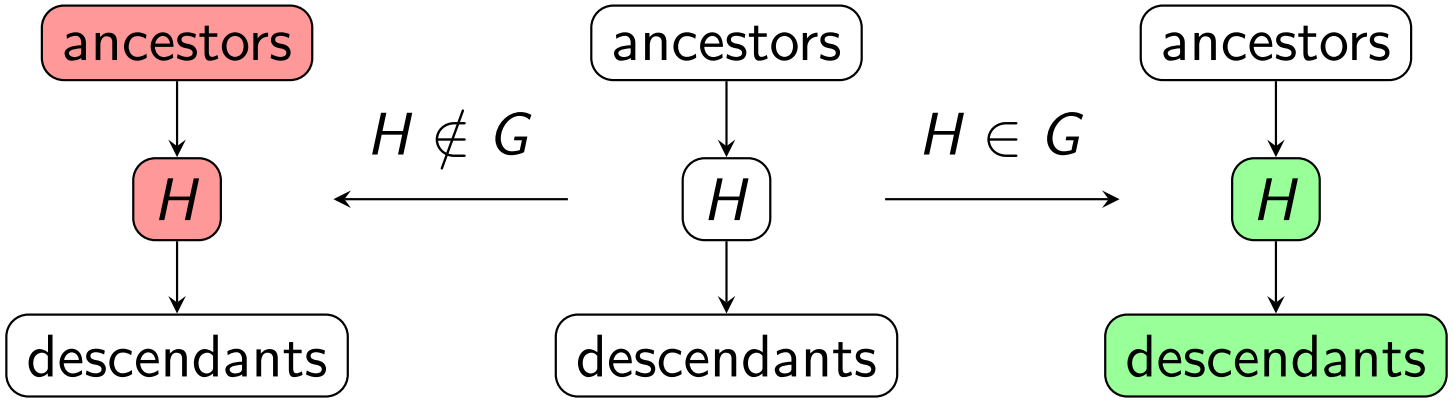

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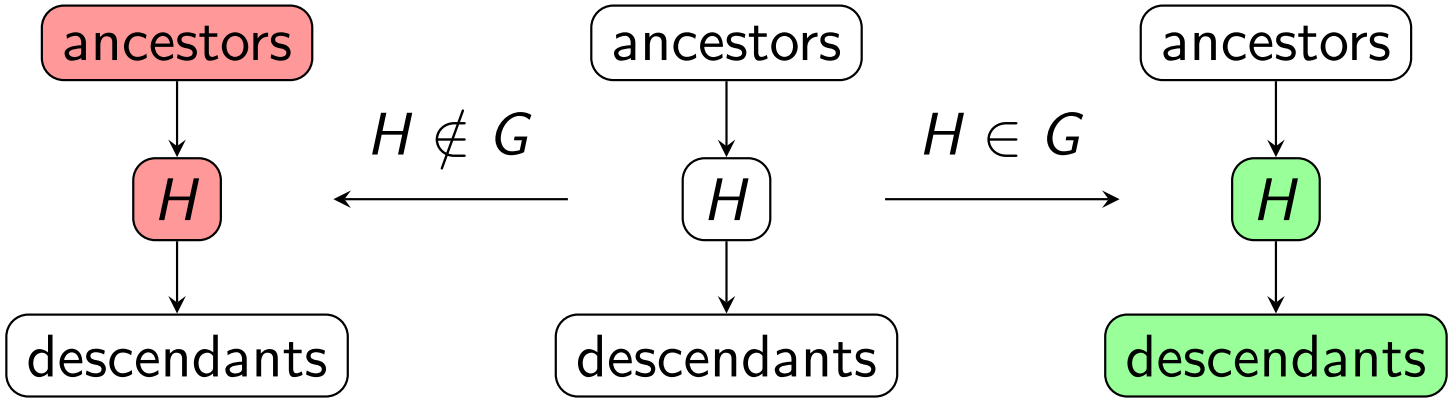

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When there's really no way around using the naïve algorithm, I launch a better tool: the Glasgow Subgraph Solver.

https://github.com/ciaranm/glasgow-subgraph-solver

Demos

Let's have a look at:

1. **single graph classification:** the Cayley graph of $\mathfrak{S}_5$ generated by prefix reversals ("pancake flipping");
2. **multiple graph classification:** all self-complementary graphs (i.e., graphs isomorphic to their complement) on 9 vertices;
3. **visualising a single graph classification:** (using external tools);

Thanks for your attention!

You can check out `graphotaxy` here:

https://github.com/alabarre/graphotaxy

Feedback is welcome and highly valued.