The Clever Shopper Problem

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Gaspard - Monge Laboratoire d'informatique

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Introduction

Clever Shopper

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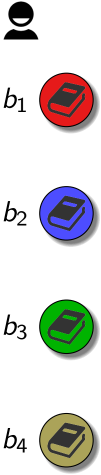

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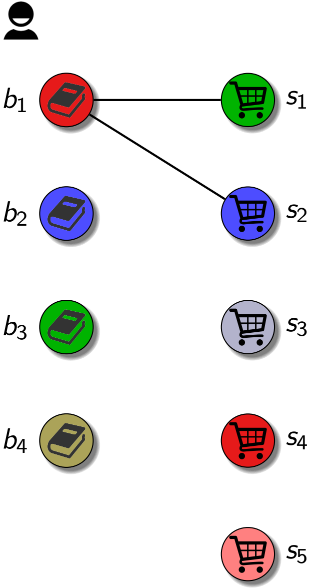

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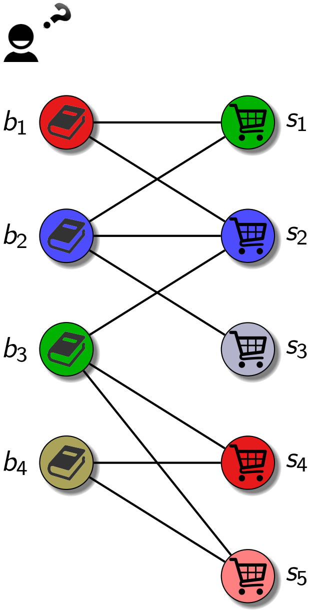

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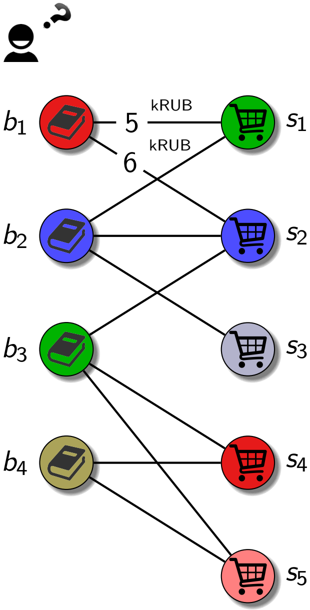

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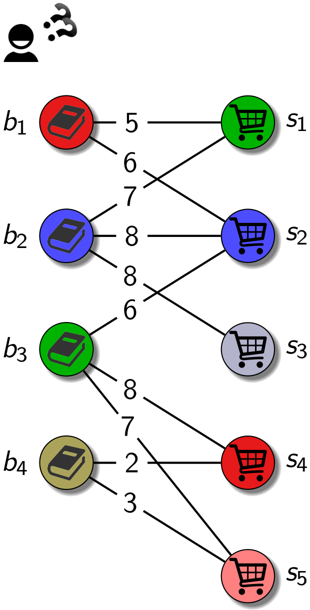

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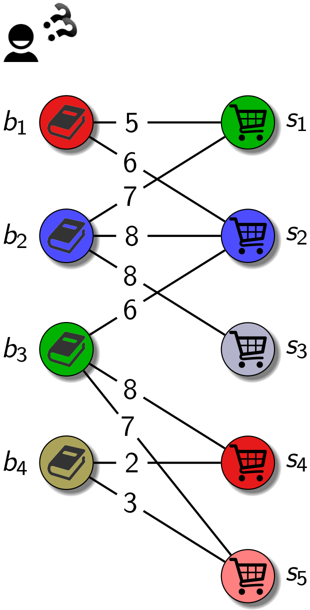

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- minimum total cost

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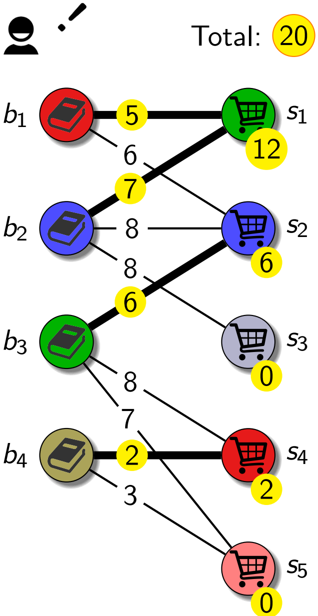

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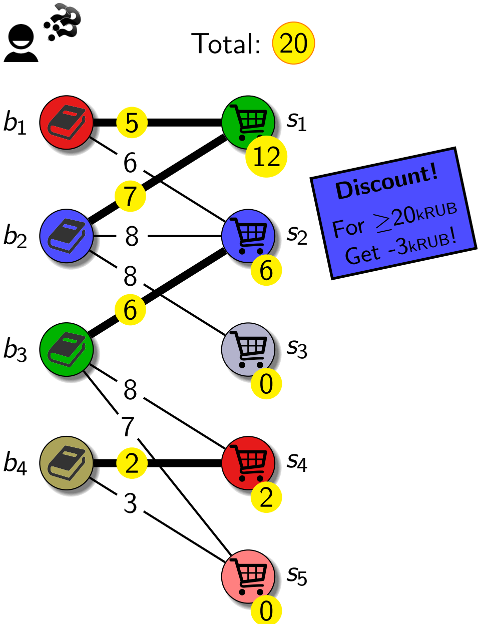

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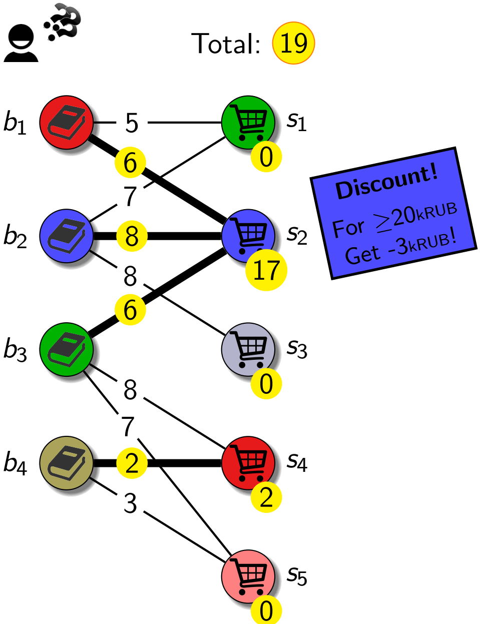

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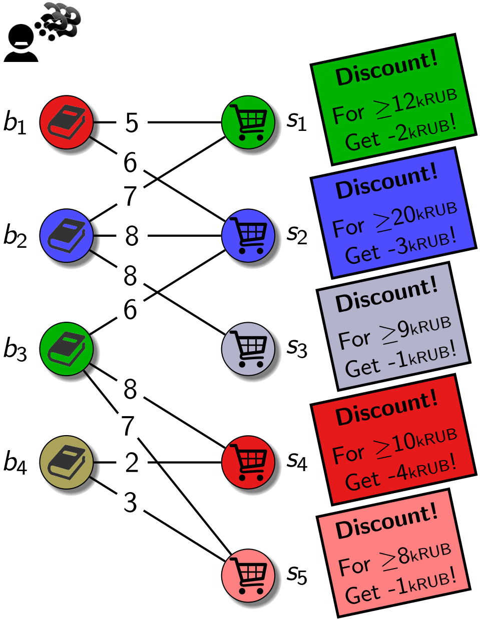

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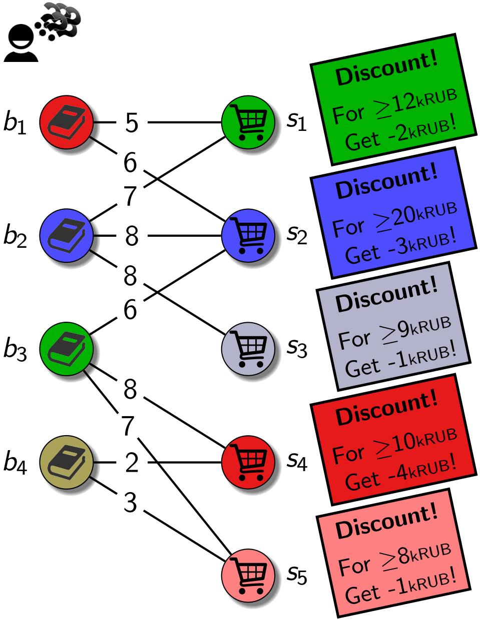

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a budget $K \in \mathbb{N}^+$

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- total price $\leq K$

Introduction

- Variant of INTERNET SHOPPING problem
  [Blazewicz et al., 2010]
  - Discounts $\leftrightarrow$ free shipping depending on specific sets of purchased items
  - Strongly NP-hard, even with free items and unit shipping costs

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  - Number of books (n)
  - Number of shops (m)
  - Price range
    Constant? Polynomially bounded? Unconstrained?
  - Degree
    Few books per shops ? Few shops selling each book?

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    Few books per shops ? Few shops selling each book?
- Any approximation algorithm?

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- NP-hard with book-degree 2, shop-degree 3 and unit prices 3-SAT

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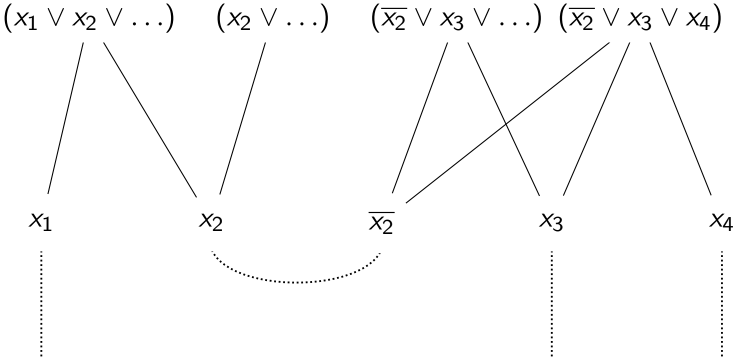
Reduction from $\textsc{Max 3-SAT}$, with 2 occurrences per literal



m clauses, n variables

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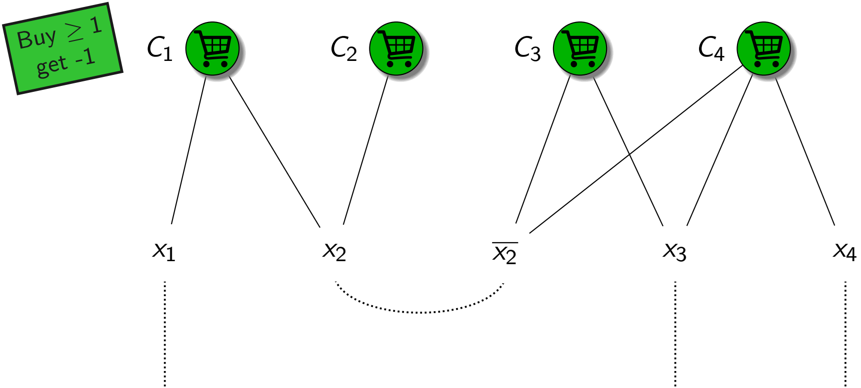

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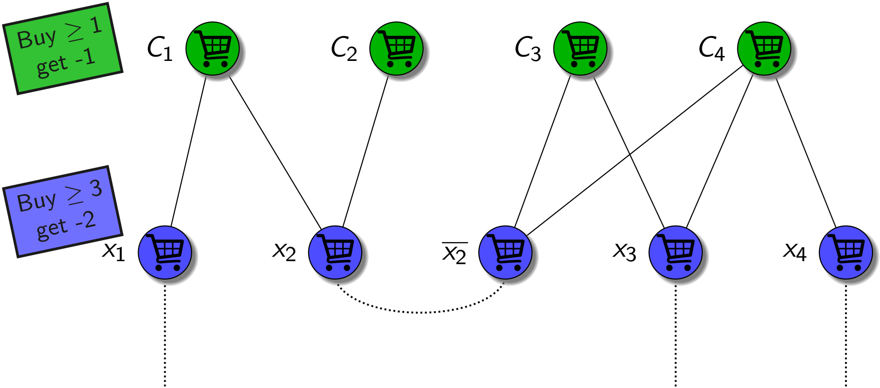

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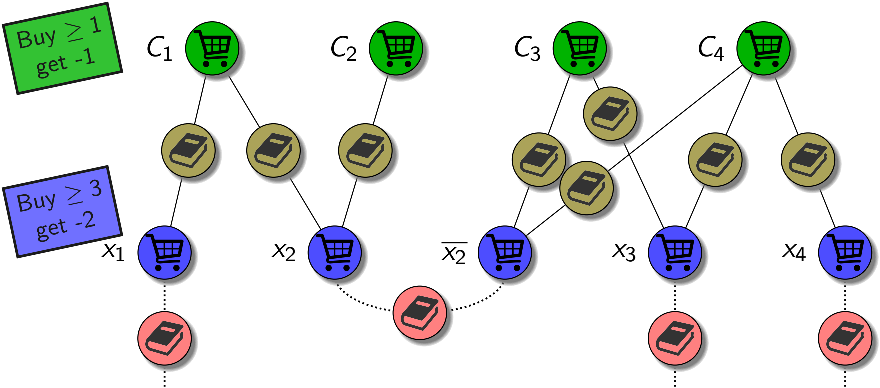

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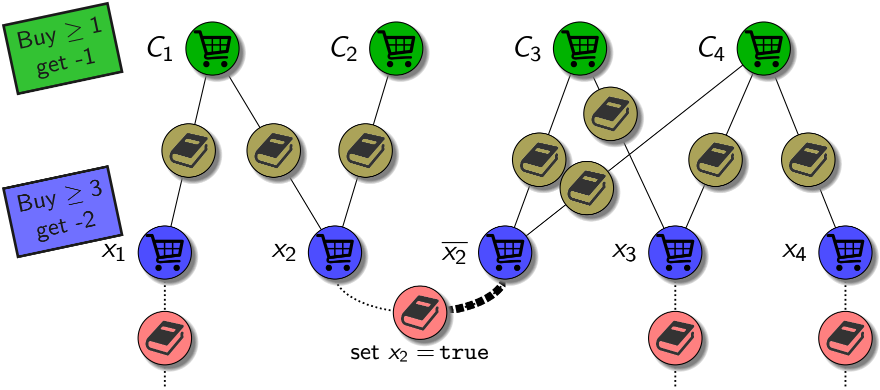

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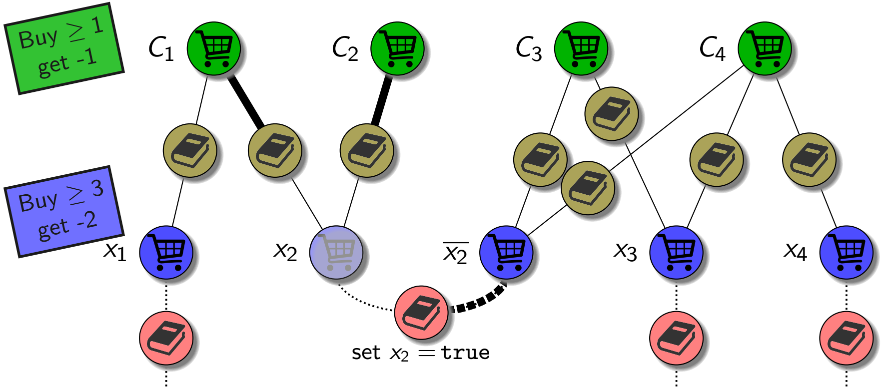

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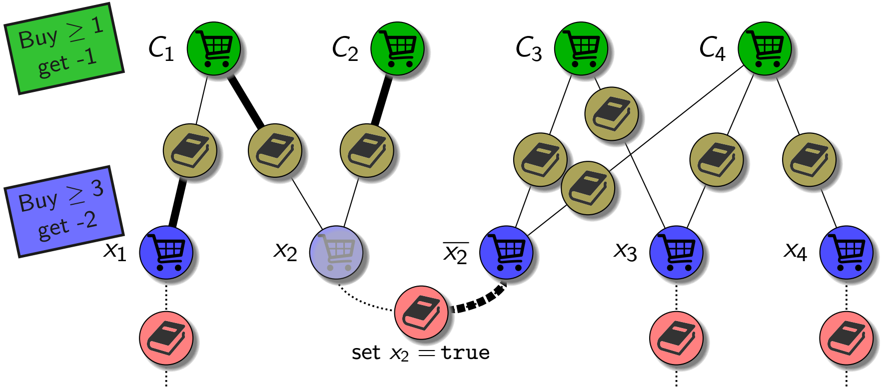

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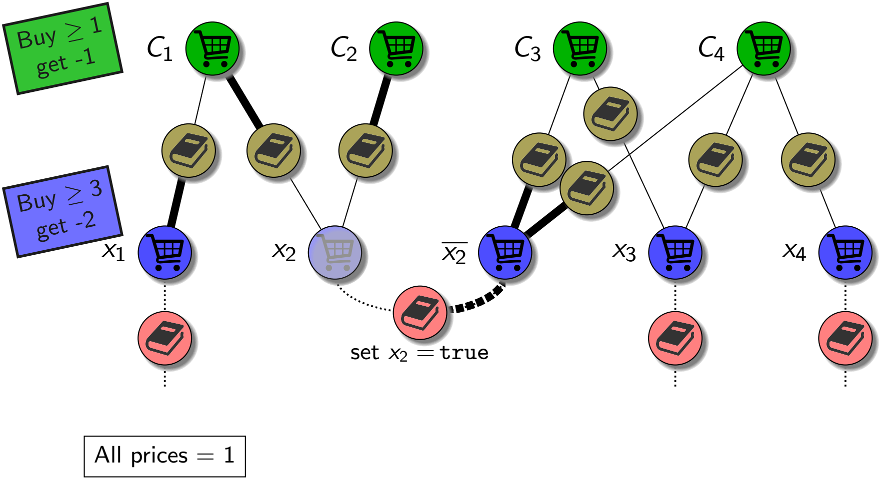

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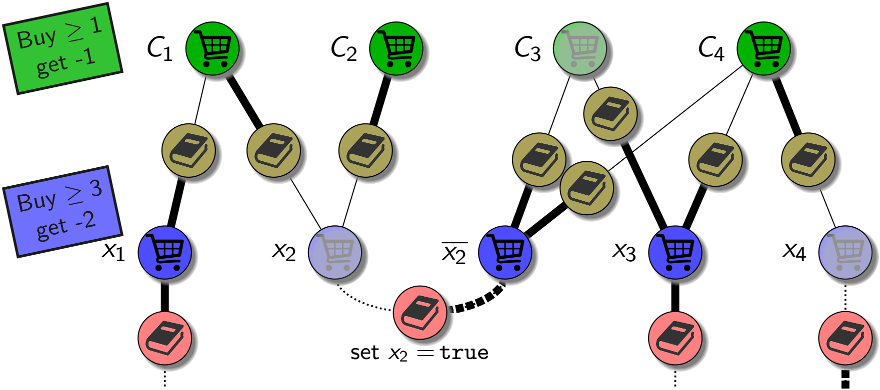


All prices = 1

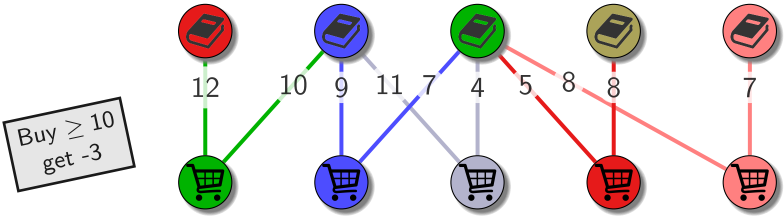
Total discount:
$2n$ + (1 per satisfied clause)

m clauses, n variables

Polynomial for shop-degree ≤ 2

Algorithm

- Subtract minimum price for each book from its incident edges



greedy solution $= (12 - 3) + 9 + 4 + 8 + 7 = 37$

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- Subtract minimum price for each book from its incident edges
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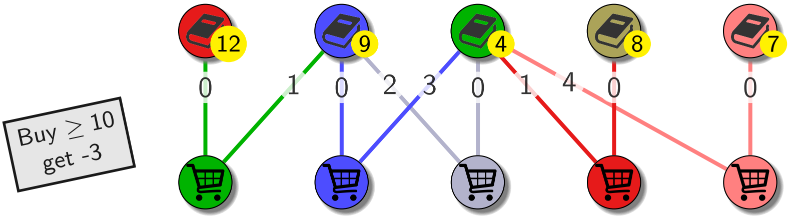

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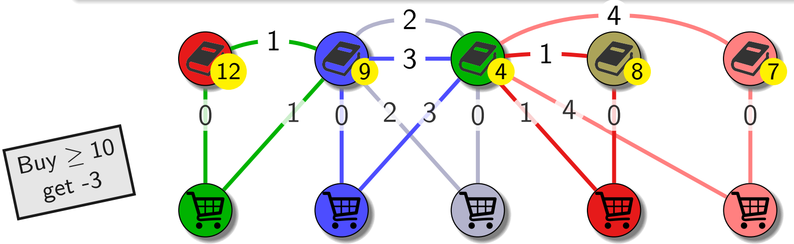
- Connect books sold by same shop with remaining cost
- Subtract discount wherever threshold is reached



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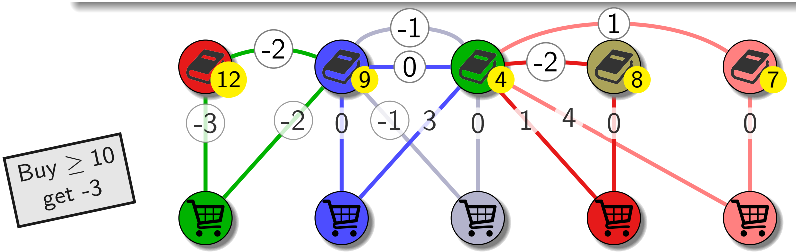

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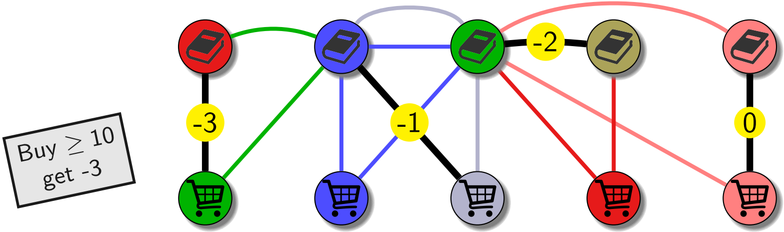

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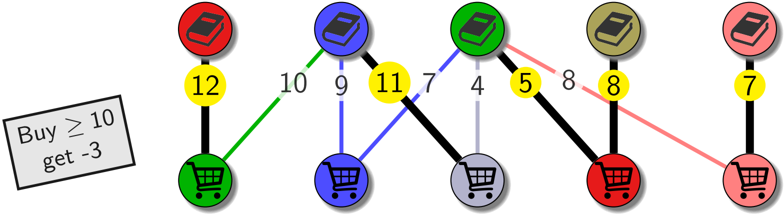

greedy solution $= (12 - 3) + 9 + 4 + 8 + 7 = 37$

optimal solution $= (12 - 3) + (11 - 3) + (5 + 8 - 3) + 7 = 34$

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Few shops (parameter *m*):

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- XP for *m* with polynomial prices — *dynamic programming*
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NP-hard with 2 shops, unbounded prices

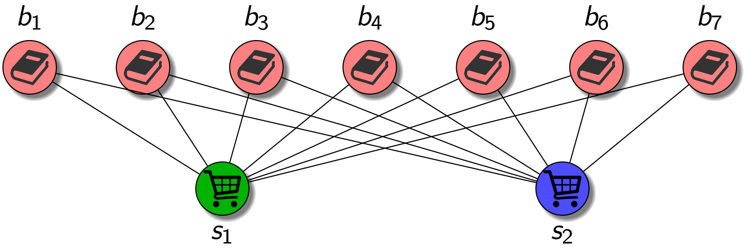
weakly NP-hard

Reduction from PARTITION

Input: $X = \{x_1, \ldots, x_n\}$ with $\sum_{x_i \in X} x_i = 2A$.

Question: $\exists ? X' \subseteq X$ such that $\sum_{x_i \in X'} x_i = A$

- 2 shops, n books
- book i has cost x_i (in both shops)
- discounts: buy A get -1
- budget: $2A - 2$



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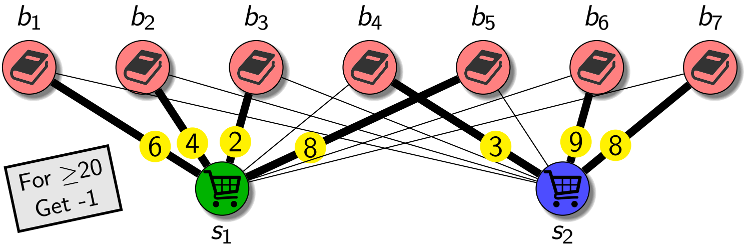

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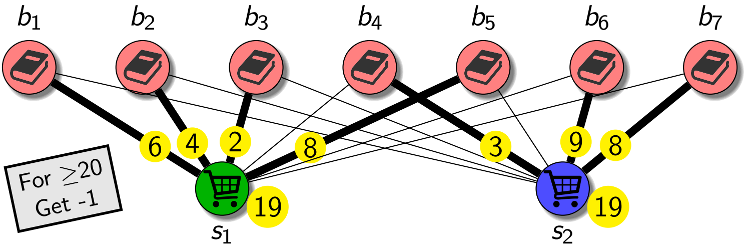

W[1]-hard for m shops, polynomial prices

Reduction from BIN-PACKING

strongly W[1]-hard

Input: $X = \{x_1, \ldots, x_n\}$ with $\sum_{x_i \in X} x_i = mB$.

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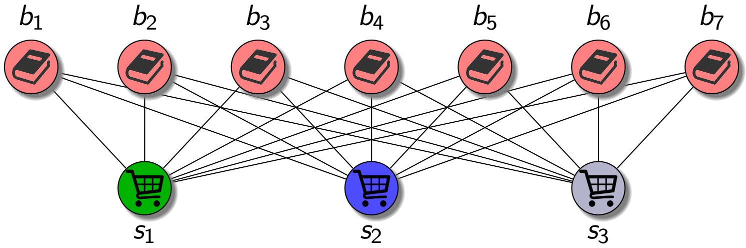

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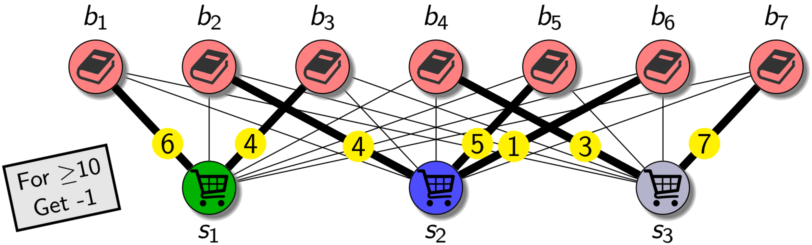

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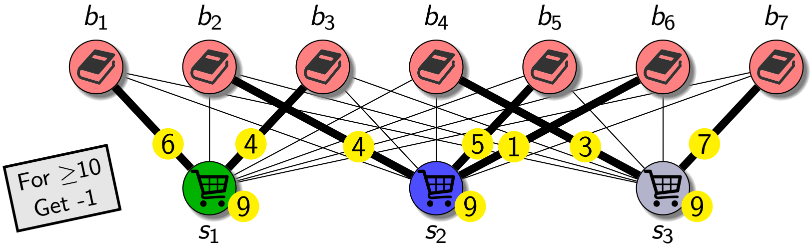

Results

Sparse instances:

- NP-hard with book-degree 2, shop-degree 3 and unit prices — 3-SAT
- Polynomial if shop degree ≤ 2 — *Matching*

Few shops (parameter m):

- NP-hard with 2 shops and unbounded prices — PARTITION
- XP for m with polynomial prices — *dynamic programming*
- W[1]-hard for m with polynomial prices — BIN-PACKING
- FPT for m with unit prices — *f-star subgraphs*
- W[1]-hard for "selected shops" with unit prices — PERFECT CODE

Approximation:

- No approximation is possible — NP-*hard with* $K = 0$
- APX-hard to maximise the total discount... — MAX 3-SAT
- ... but k-approximable for shop-degree k — *greedy*

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Other optimisation strategy: maximise the total discount

- Meaningful only if each book has a uniform price
- Max 3-SAT reduction yields APX-hardness.

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FPT for number of books n

Dynamic Programming Table:

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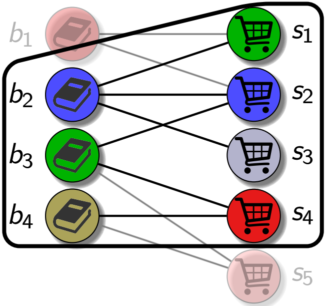

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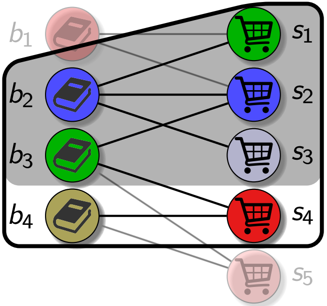

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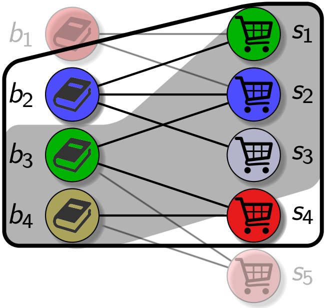

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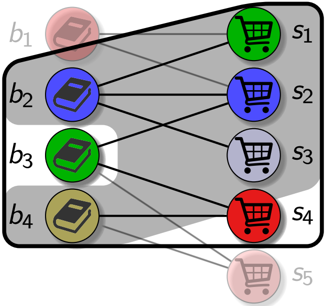

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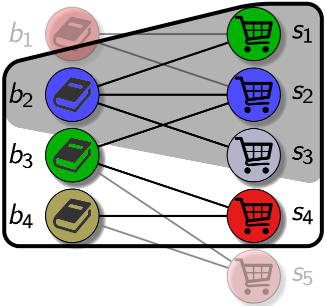

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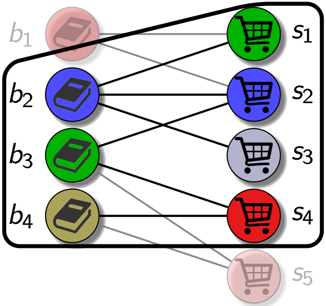

For each j: enumerate every $B'' \subseteq B' \subseteq B$

$\longrightarrow \mathcal{O}(m3^n)$

?? Open questions

- Constant-factor approximation (maximising total discount)?
- Kernel for parameter m with unit prices?
- FPT for number of shops + max. price?
- What if all books are available everywhere at constant price?

Thank you!