

Codes and Automata

Corrections and Complements

June 12, 2011

This file contains corrections and complements to the book.

1 Preliminaries

- p. 28 *l.* -2 : Insert ‘provided the automaton is complete’ after ‘The matrix M/k is stochastic’.
- p. 37 *l.* 2 of proof of Proposition 1.10.10 : remove the last ‘×’.

2 Codes

- p. 74 *l.* 16 : Insert ‘ $= pqt^2 F_{D_a^*}(t)$ ’ after ‘ $F_a(t)F_{D_a^*}(t)F_b(t)$ ’
- p. 102 *l.* 3 of Exercise 2.4.2 : Replace ‘prefix of w ’ by ‘prefix u of w ’

3 Prefix codes

- p. 114 Figure 3.8(b) : Replace the label ‘ a ’ by ‘ b ’ on the last edge of the path of length 3.
- p. 117 *l.* -8 : Replace ‘minimal automata’ by ‘minimal automaton’
- p. 157 *l.* -7 : Replace ‘ \mathcal{B} ’ by ‘ \mathcal{B} of the proof of Lemma 3.8.6’
- p. 173 Exercise 3.8.2 *l.* 1 : Add ‘s’ to ‘length’
- p. 173 Exercise 3.8.2 *l.* 3 : Insert ‘3.8.1 and’ before ‘3.6.4’

4 Automata

- p. 194 Example 4.3.5 : ‘the code $X =$ ’ instead of ‘the code $C =$ ’

5 Deciphering delay

- p. 214 *l.* 15 : Insert ‘with $a \in A$ ’ at the beginning of the line

6 Bifix codes

- p. 227 *l.* 11 : ‘Proposition’ instead of ‘Theorem’
- p. 229 *l.* 9 : ‘any parse of v ’ instead of ‘any parse of u ’
- p. 230 *l.* 2 : Replace ‘Theorem 3.1.6’ by ‘Proposition 3.1.3’, and insert ‘by Proposition 3.1.6’ before ‘ $1 - \underline{X}$ ’.
- p. 233 *l.* 5 : ‘for $k = 0, 1$ ’ instead of ‘for $k = 0, 1, 2$ ’

- p. 234 $\ell.$ -8 : ‘Corollary’ instead of ‘Proposition’
- p. 245 $\ell.$ 2 of Proposition 6.3.14 : ‘ $H = A^-XA^-$ ’ instead of ‘ $H = A^* \setminus XA^-$ ’
- p. 274 $\ell.$ 7 : Insert ‘Exercise 6.1.2 is from Reutenauer (1979)’

10 Synchronization

- p. 395 Section 10.6 Notes : Insert ‘The notion of *constant* appears in Schützenberger (1975). The notion of *synchronizing word* appears in many contexts with various denominations, including magic word (Lind, Marcus (1995)) or reset sequence. It has been defined in Chapter 3 for prefix codes and for deterministic automata. The notion of *synchronizing pair* is an extension of the definition of synchronizing word to codes which are not prefix. It is due to Schützenberger (1979b).’
- p. 395 Section 10.6 Notes $\ell.$ 3 : Insert before ‘However’ the sentence ‘This is Theorem 10.2.11.’

11 Groups of codes

- p. 415 Remark 11.4.5, $\ell.$ 3 : Insert a space between ‘ X ’ and ‘is’
- p. 495 proof of Proposition 14.1.2, $\ell.$ 1 : Replace ‘Let X, Y ’ by ‘Let X, Z ’

13 Densities

- p. 452, $\ell.$ 1 : Replace the first paragraph by:
A real valued function μ defined on a Boolean algebra of sets \mathcal{F} is *additive* if for any disjoint sets $E, F \in \mathcal{F}$, one has $\mu(E \cup F) = \mu(E) + \mu(F)$. It is called *countably additive* if

$$\mu\left(\bigcup_{n \geq 0} E_n\right) = \sum_{n \geq 0} \mu(E_n)$$

for any sequence $(E_n)_{n \geq 0}$ of pairwise disjoint sets in \mathcal{F} such that $\bigcup_{n \geq 0} E_n \in \mathcal{F}$. If μ is additive and takes nonnegative values, then it is *monotone* in the sense that if $E \subset F$ for $E, F \in \mathcal{F}$, then $\mu(E) \leq \mu(F)$ since indeed $\mu(F) = \mu(E \cup (F \setminus E)) = \mu(E) + \mu(F \setminus E) \geq \mu(E)$.

- p. 452, $\ell.$ 8 : Replace Proposition 13.1.3 by:
Let μ be a countably additive function defined on a Boolean algebra \mathcal{F} of sets. Then

$$\mu\left(\bigcup_{n \geq 0} E_n\right) \leq \sum_{n \geq 0} \mu(E_n)$$

for any sequence $(E_n)_{n \geq 0}$ of sets in \mathcal{F} such that $\bigcup_{n \geq 0} E_n \in \mathcal{F}$.

- p. 456 : Replace Proposition 13.1.13 by:
The function μ satisfies $\mu(A^\omega) = 1$ and is countably additive.
- p. 457 $\ell.$ 8 : Add ‘The second inequality holds by Proposition 13.1.3 since, by Lemma 13.1.12, \mathcal{F} is a Boolean algebra.’

- p. 457 *l.* 10 : Replace the sentence by: A function ν defined on a family of sets \mathcal{F} is called *countably subadditive* if for any sequence $(E_n)_{n \geq 0}$ of sets in \mathcal{F} such that $\bigcup_{n \geq 0} E_n \in \mathcal{F}$, one has $\nu(\bigcup_{n \geq 0} E_n) \leq \sum_{n \geq 0} \nu(E_n)$.
- p. 459 *l.* 15 : Replace ‘Then Equation (13.1) holds’ by ‘Then the equation of line 6 holds’

14 Polynomials of finite codes

- p. 525, *l.* -6 : Replace ‘ $\tau m = \tau m + \tau' m$ ’ by ‘ $\sigma m = \tau m + \tau' m$ ’
- p. 526 Example 14.7.3 : The matrices α and β should be

$$\alpha = \left[\begin{array}{c|ccc} 1 & 0 & 0 & 0 \\ \hline 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \end{array} \right], \quad \beta = \left[\begin{array}{c|ccc} 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & -1 \\ 0 & -1 & 1 & -2 \\ 0 & -1 & 1 & -2 \end{array} \right].$$

Solution of exercises

- p. 544 Solution 3.8.1 : *l.* 3 of p. 544 : Replace ‘ $v_{n+p} = v_n k^p - \sum_{i=1}^p u_{n+i} k^{p-i}$ ’ by ‘ $v_n = k^p - \sum_{i=1}^n u_{n-i} k^i$ ’, and insert *l.* 4, before ‘Using’ the sentence ‘It implies that $v_{n+p} = v_n k^p - \sum_{i=1}^p u_{n+i} k^{p-i}$.’
- p. 552 Solution 6.1.2 : replace lines 5–9 by ‘Next, if (ii) holds, consider $x \in H \cap A^*$. Then $x = h_1^{\epsilon_1} h_2^{\epsilon_2} \cdots h_n^{\epsilon_n}$ with $n \geq 0$, $h_i \in X$ and $\epsilon_i = \pm 1$. We may assume that n is chosen minimal. Assume that $\epsilon_i = -1$ for some index i . Since X is bifix, none of the h_i^{-1} can cancel completely with h_{i-1} or with h_{i+1} . Since $x \in A^*$, there exists an index i with $1 \leq i \leq n$ such that $\epsilon_i = -1$ and $h_i^{\epsilon_i}$ cancels with its neighbors, that is $h_{i-1}^{\epsilon_{i-1}} h_i^{\epsilon_i} h_{i+1}^{\epsilon_{i+1}} \in A^*$. Thus, we have $\epsilon_{i-1} = 1$, $\epsilon_{i+1} = 1$ and $h_{i-1} = tu$, $h_i = vu$, $h_{i+1} = vw$ for $t, u, v, w \in A^*$. But then $h_{i-1} h_i^{-1} h_{i+1} = tw$ is in X by (ii). This contradicts the minimality of n . This shows that $\epsilon_i = 1$ for all i and thus $x \in X^*$. Thus (iii) holds.’
- p. 584 Solution 14.1.3 : insert ‘strict’ before ‘right contexts’ and ‘left contexts’

Appendix: Research problems

- p. 593 *l.* 8 : Replace the last sentence of the paragraph by ‘It is conjectured that for any finite maximal prefix code X there exist $P, T \subset A^*$ such that

$$\underline{X} - 1 = \underline{P}(A - 1)\underline{T}$$

where T is the union of $d(X)$ pairwise disjoint maximal prefix sets (see Perrin, Schützenberger (1992)). This is equivalent to say that in Equation (14.7) one has $S = 1$ and the polynomial Q has the form $Q = \sum_{i=1}^{d-1} \underline{U}_i$ where each U_i is a nonempty prefix-closed set.’

- p. 593 *l.* -4 : Replace ‘finite set Y ’ by ‘finite subset Y ’

References

- p. 596 *l.* -10 : Replace ‘Capoceli’ by ‘Capocelli’

Index

- p. 616 *l.* 5 : Replace ‘nil-simple semigroup 417’ by ‘nil-simple semigroup 416’

Additional reference

Christophe Reutenauer (1979). Une topologie du monoïde libre. *Semigroup Forum*, **18**(1):33–49, 1979.