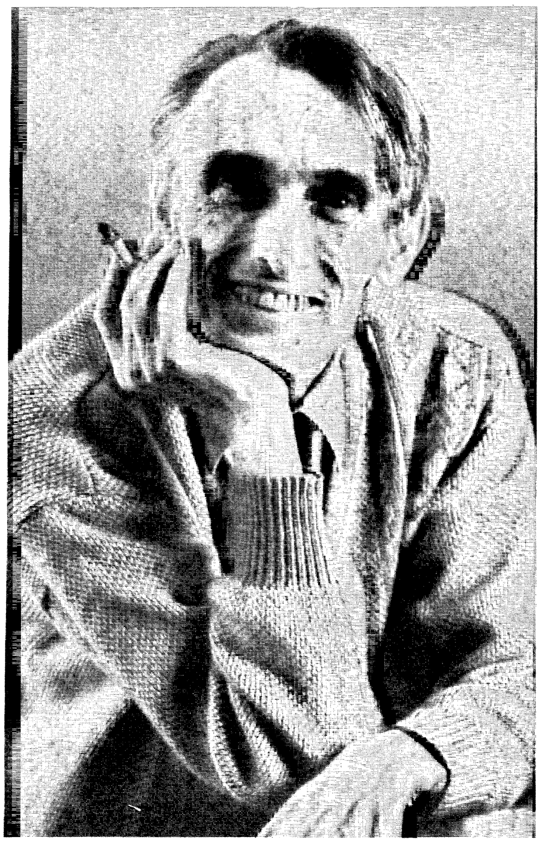
Semigroup Forum Vol. 55 (1997) 135–151



Marcel-Paul Schützenberger
1920–1996

OBITUARY

Marcel-Paul Schützenberger (1920–1996)

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Communicated by John M. Howie

Marcel-Paul Schützenberger died in Paris on July 29, 1996. He was one of the most prolific and original discoverers of the twentieth century having worked in areas as diverse as genetics, statistics, information theory, combinatorics, coding and automata theory, the theory of formal languages, and on various problems related to linear representations of the symmetric group.

Inspired by deep problems from information theory, linguistics, biology, his work in theoretical computer science is of fundamental importance and early on in his life he had been recognized as one of the few worldwide creators of this area of scientific endeavor which he turned into an integral part of mathematics. His contributions to combinatorics are just as deep and important as those in computer science making him, to quote Herbert Wilf, "one of the most creative and influential combinatorialists of this century". But foremost he was an outstanding mathematician and the following statement, taken from an unpublished survey of his work which he wrote almost ten years ago, summarizes nicely his motivations and philosophy:

> *"Ma conception des mathématiques appliquées est de voir dans les applications une source extérieure de théorèmes et d'intuition de technique de preuves pouvant être utilisées pour le progrès de la science mathématique en supplément des motivations internes que fournit la structure et le mouvement mêmes de cette science."*

M. P. Schützenberger was born in Paris on October 24, 1920 in a family of Alsatian origin; his great grand-father had moved from Alsace to Paris a few years before the 1870–1871 Franco-Prussian war. His university studies were somewhat disrupted by the second world war; he served from September 1943 to August 1944 in the "Forces Françaises Combattantes". Resuming then his studies at the Faculty of Medicine of Paris, he became a Doctor in Medicine in 1948. From 1948 to 1953 he was a researcher at the National Institute of Hygiene working on various problems in genetics (e.g. he contributed to the discovery of the gene of trisomy) and in the excitation of the senses, pioneering the use of statistical methods in these and other areas of medicine. At the same time he was already involved in various mathematical questions related to lattices, statistics, block-designs and other combinatorial problems, culminating in his doctoral thesis in 1953 which expanded vigorously on C. Shannon's 1949 basic paper on the mathematical theory of communications. The same year he obtained a position of Maître de Recherche at the C.N.R.S. (National Center of Scientific Research) continuing his previous work on combinatorics, communications, and more and more frequently on semigroups, automata, and codes. In

1956 he was invited to spend a year at the Research Laboratory of Electronics at the M.I.T., to be part of Shannon's research team. Returning to France the following year he took up a position at the University of Poitiers where he taught statistics until 1963. After one year spent as Research Director at the C.N.R.S. he became Professor at the Faculty of Sciences of the Sorbonne in 1964 and from 1968 to 1972 served simultaneously as Scientific Director at the I.R.I.A. (Research Institute in Information and Automata). The period 1959–1966 was marked by his numerous visits to American universities, notably M.I.T., North Carolina, Harvard, Pennsylvania, Berkeley, as well as consulting work with I.B.M. and the Rand Corporation. His visit at the University of Naples in 1972–73 saw the birth of a group of very active Italian disciples, that has worked in close collaboration with the group in Paris, and has constantly grown in number since that time.

From 1969 to 1980 M. P. Schützenberger also served as Scientific Counselor to the General Director of the World Health Organization participating in the set up of international teams of experts to study various problems like the prevention of the use of chemical and biological weapons or the establishment of a worldwide network to detect or prevent widespread accidents due to the careless use of medication. In 1979 he became a corresponding member of the French Academy of Sciences and was elected to full membership in 1988.

Among the many algebraic structures that appear in Schützenberger's works a dominant one is the structure of semigroup. Semigroups appear in his works essentially in two forms: 1) as free monoids, for example in coding and language theories and often in combinatorial designs; 2) as finite semigroups, for example in transition semigroups of automata or in the theory of pseudo-varieties for the study of varieties of languages. On finite semigroups he wrote the following lines:

> *"Les semigroupes finis ont, je crois, un rôle à jouer dans la pratique et l'enseignement de demain car, sans même parler de l'informatique, la technique contemporaine avec ses multiples systèmes digitaux abonde en situations où l'on a besoin de résultats assez fins concernant le semigroupe engendré par des actions opérant sur un ensemble fini. La théorie des semigroupes finis n'est ni triviale, ni formelle et ne constitue en rien une variante ou une généralisation de celle des groupes ou des anneaux. Elle est indiscutablement beaucoup moins riche que ces dernières mais ses théorèmes fondamentaux (de Suschkevitch, de Greene, de Clifford) mériteraient d'être plus largement connus. Il ne s'agit pas d'élargir ou d'approfondir des galeries dans la mine d'or de la théorie des groupes mais d'inventer quasiment de toutes pièces des énoncés et des techniques de preuve et des objets nouveaux pour appréhender les systèmes discrets finis. La théorie des codes et celle des automates finis proposent des problèmes précis et parfois suggèrent des solutions. Au bout de nos efforts nous espérons trouver parfois quelque chose à apporter au courant principal de la Mathématique."*

This last sentence is important and it is true indeed that Schützenberger's works contributed strongly to giving to semigroup theory its letters of nobility by taking it out of its self-contemplation and putting it at work in areas of mainstream mathematics. The purpose of the following analysis of some of his work is to show how he and his followers have achieved this.

One of the most important creations of Schützenberger is what is now known as the theory of variable-length codes. In 1955 he presented a paper entitled "Une théorie algèbrique du codage" at the algebra seminar held at the Institut H. Poincaré in Paris. This paper which defines a code as the base C of a free submonoid C^* of the free monoid A^*, making it a perfectly natural algebraic object, contains also many of the basic ideas of his work in automata theory. For example it contains the definition of the syntactic monoid of a subset of A^*, the notion of recognizable sets and their coincidence with rational sets. Kleene's work on this appeared in "Automata Studies" (Princeton, 1956) using the less transparent concept of abstract nerve net.

He observed that complete prefix codes (i.e., bases of stabilizers of a state in a deterministic automaton) were related to some stochastic processes introduced by W. Feller called recurrent events. This led him naturally to use methods of probability theory in the study of codes, along with combinatorial properties of words and algebraic properties of the corresponding syntactic monoids. This interplay between algebra, probability and combinatorics built up a fascinating and rich theory which often yields surprising connections to other central areas of mathematics. For example he showed that some synchronous codes are in one-one correspondence with bases of the free Lie algebra, thus allowing an elegant proof of the well-known Poincaré-Birkhoff-Witt theorem via factorizations of free monoids. Many of the further works on codes were stimulated by his conjectures, and most of the beautiful results of the theory are presented in the book by J. Berstel and D. Perrin "Theory of Codes" (Academic Press, 1985).

Schützenberger started to work on context-free languages in the late fifties. One of his most remarkable ideas was the use of power-series in non-commutative variables with integral coefficients. In this approach a context-free language appears as the support of the power-series solution of systems of equations directly related to the usual production rules of the language. The so-called regular languages are obtained this way with equations where the unknowns appear in monomials of degree 1. The corresponding power-series are the analogs of rational power-series in one variable. Those corresponding to context-free languages are the analogs of algebraic power-series. This is ultimately the most elegant approach to the study of context-free and regular languages, rebaptized more appropriately as algebraic and rational languages. It also has the effect of putting the theory of these languages right back within the framework of classical mathematics. For example, in this context, the theorem that the intersection of an algebraic and a rational language is algebraic appears as the non-commutative generalization of a theorem of Jungen on the Hadamard product of an algebraic and a rational power-series in one variable.

In 1963, jointly with N. Chomsky, he published "The algebraic theory of context-free languages" which contains the famous Chomsky-Schützenberger theorem stating that algebraic languages can be characterized as those obtainable by homomorphic images from languages of the form $D_n^* \cap K$ where K is rational and D_n^* is the Dyck language (i.e. the class of 1 of the congruence generated by $x_i\overline{x}_i \sim \overline{x}_i x_i \sim 1$ for $1 \leq i \leq n$), thereby providing a surprising appearance of the free group in the theory. The very simple algebraic form of this result allows for frequent simplifications of proofs which otherwise would have to use less smooth push-down automata techniques.

One of the most beautiful works of Schützenberger was inspired by the work of R. McNaughton who showed that a certain first order propositional calculus was in one-one correspondence with subsets of the free monoid obtainable with Kleene's operations except that the star operation must be replaced by complementation. These are called star-free sets. The problem was: Is the property for a rational set of being star-free decidable? Schützenberger's positive answer appeared in 1965. A rational set is star-free if and only if its syntactic monoid has only trivial subgroups (such semigroups are now called aperiodic semigroups). This was the crucial result that led him to develop, jointly with S. Eilenberg, the theory of pseudo-varieties of semigroups characterized by ultimate equations. Many researchers have continued to investigate this type of algorithm for deciding whether or not languages belong to diverse varieties, bulding up a theory with ingenious algebraic and combinatorial tools, deep problems and precise conjectures (see S. Eilenberg's "Automata Languages, Machines" Vol. B, 1976 or J. E. Pin's more recent "Varieties of Formal Languages").

Part of the proof he gave of the star-free theorem is based on a deep knowledge of the Green structure of finite semigroups. In fact he had contributed earlier to the Green theory by showing that to any $\mathcal{D}$-class D of a semigroup S one could associate a group G acting transitively on each $\mathcal{H}$-class in D, and a representation of S by matrices with entries in $G \cup \{0\}$. These were called Schützenberger groups and representations by A. H. Clifford and G. B. Preston. This work on semigroups which has important applications (e.g. to the study of homomorphisms, in particular linear representations) is among the few that do not seem to have been directly motivated by applied considerations but rather by a need to understand the fine structure of semigroups.

Another area of theoretical computer science where he made important contributions is the theory of transducers. These are finite automata whose transition edges are labeled by pairs of words from an input free monoid X^* and an output free monoid Y^*. The simplest of these are the sequential transducers where output words are stretched step by step (for example on the right) by words depending only on the letter of X and the state to which this letter is applied. Schützenberger studied these and introduced what is called bimachines in Eilenberg's book; these can be simulated by two sequential transducers reading respectively left to right and right to left. He showed that these bimachines realize all rational partial functions from X^* to Y^* (i.e., all functions whose graphs are rational subsets of $X^* \times Y^*$). Here again the motivation for the introduction of bimachines was the decoding of messages using general variable-length codes, and readers familiar with the standard decoding algorithms can appreciate the much more dynamic aspect of this approach to the decoding problem.

Throughout practically all his works some of the most basic tools used are properties of words. In his mind the free monoid on an alphabet with n letters is the non-commutative generalization of $\mathbf{N}^0$. The study of the latter gives rise to (additive) number theory, the study of the former gives rise to what is called combinatorics on words. A more revealing name might perhaps be non-commutative number theory.

The very first book on this subject appeared in 1983; it is the collective work of Schützenberger and some of his closest disciples and the chosen name for the author of the book is M. Lothaire* (Combinatorics on words, Encyclopedia of Mathematics and its Applications, vol 17). The book arose from a text of lectures he gave in Paris in 1966 "Quelques problèmes combinatoires de la théorie des automates" which served to many people as a fondly remembered introduction to the field. As a tribute to his perception of the existence of this form of non-commutative number theory some of his friends and disciples offered him a collection of articles, many of which are related to his work, in a book, again authored by M. Lothaire, entitled simply "Mots" (Hermes, Paris, 1990).

The importance of words in his opus is also transparent in his contributions to combinatorics. For example in 1970, jointly with D. Foata he published a monograph studying the Eulerian polynomials of Frobenius. They show that many of the formulas related to these polynomials can be established by constructions on permutations viewed as words with non-repeated letters. The general idea of representing collections of combinatorial objects by words or non-commutative power-series has proven to be a very successful method in further studies on hypergeometric series and generalized orthogonal polynomials.

Schützenberger's work on combinatorial questions related to linear representations of the symmetric group S_n started in 1963 by a study of an algorithm of C. Schensted establishing a bijection between permutations and pairs of standard Young tableaux. In particular he systematized the transformation of properties of tableaux into properties of permutations (again often viewed as words). This work was resumed and extended in a paper of 1977 on the G. de B. Robinson correspondence which was an earlier form of Schensted's algorithm. This paper contains one of the first correct proofs of a central and most difficult result of the theory, the Littlewood-Richardson rule. It also contains a most useful tool for the study of tableaux, the plactic monoid (quotient of the free monoid admitting the tableaux as cross-section). His collaboration with A. Lascoux in this area goes back to 1978 and continued until the very last days of his life. It produced an impressive number of deep results solving problems of Kazhdan and Lusztig, a conjecture of Foulkes on polynomials that yield the characters of S_n in characteristic q, giving new interpretations of generalized Schur functions, introducing Schubert polynomials and the tableaux algebra related to the game of "taquin", leading progressively to a deeper understanding and a more unified treatment of all these questions.

These are amazing achievements of an intellectual giant of our time. They are the productions of a man who, most of his life and until the very end of it, was passionately involved in mathematics, in sciences in general, and in all kinds of human activities. He always held strong opinions — sometime contradictory as his life evolved — on a large variety of topics as diverse as the Darwinian theory of

(*) Lothaire is the name of one of the three grand-sons of Charlemagne. By the treaty of Verdun in 843 he received a part of Charlemagne's empire whose width is approximately that of Alsace-Lorraine but which extended from the North Sea to way down in the middle of Italy. This region called Lotharingia is related to the origins of Schützenberger's family and to that of some of his mathematical descendants. The University of Strasbourg offers on a regular basis a Lotharingian Seminar of Combinatorics.

evolution (which he found incompatible with any serious statistical analysis) or the reform of the spelling of the French language ("don't touch my cedilla!"). His was a complex and unorthodox personality. He was witty, humorous, had a passion for discussion, if not controversy, an immoderate love for paradoxes and was capable of the most laudatory praise as well as the most ironic sarcasm. His lectures, reflecting this personality, were often difficult to follow. Many of his listeners would after a lecture ask themselves "What does he see that I cannot see yet?" To some, this was strong motivation to find out and be awed by what there was to be seen. The influence he had on his students went well beyond their mathematical education. To all he was a close and affectionate friend, and to his friends a man of an incredibly generous nature. His lifetime achievements have been of the first magnitude, and their consequences will keep him very much alive in the memories of future generations of researchers.

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Received April 14, 1997