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A TENTATIVE CLASSIFICATION OF GOAL-SEEKING BEHAVIOURS.*

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Goal-seeking behaviour, once a great mystery, is now beginning to be understood. In its simplest forms it is, in fact, understood to-day almost completely. Thus the theory of the simple regulator, such as the thermostat, not only includes an extensive repertoire of techniques, but the elementary principles, of the necessity for negative feedback for instance, are becoming scientific commonplaces. In getting to know, however, about these simple systems and their principles, we should not make the mistake of thinking that there is nothing more to be learned. On the contrary, in real life many an important goal is to be achieved only through some quite complex pattern of behaviour, a pattern for which the simple concept of "negative feedback" is quite inadequate. It is of these more complex patterns that I wish to speak to-day.

One way of studying the subject is by way of actual experiment; but I shall make little reference to actual experiments to-day. The fact is that before such experimentation can be undertaken with any usefulness there must be a preliminary period of study and thought. Before we can experiment we must be clear about what questions we want to ask, and why these questions are significant, and what are to be the interpretations of the experiment's various possible outcomes. Before we can usefully start experimenting, in other words, we must have a well-developed theory. Such a theory must inevitably, if it is to be precise, be mathematical; but I hope to show in this paper that what is necessary, at least at first, is the logic and precision of mathematical thought rather than its more advanced techniques. If then we are to explore the properties of the more complex forms of goal-seeking behaviour we must first construct some suitable mathematical models.

The use of mathematical models in the study of animal and human behaviour goes back to the early nineteenth century, but those early attempts have little in common with the contemporary researches of such scientists as von Neumann, Wiener, and, here among us, Dr. Ashby. The last author has particularly stressed the similarity between the activities of some types of mechanism and that of the brain; and this similarity must be the excuse for a mathematician such as myself venturing into this controversial field. My aim will be to show how certain of the more complex goal-seeking behaviours, seen in both machines and men, can be described, and the principles made clear, by a uniform mathematical framework of ideas.

* A paper read at the Annual Meeting of the Royal Medico-Psychological Association held at Barnwood House, Gloucester, 9 July, 1953.

Basic Concepts.

In order to make the ideas as clear as possible I shall start with a very simple example. Let us suppose a man is on the top of a hill and that he wishes to get to a house in the valley; let us assume that the "goal" is his arrival there in *the shortest possible time*. Between him and the house are many causes of delay: boulders, marshes, escarpments, and so on. Travelling in a bee-line is out of the question. Let us consider his possible modes of behaviour.

An exhaustive, and final, solution of the problem would be given by taking a map of the district, dividing it into small areas, finding the time taken to cross each area individually, joining the areas into all possible chains between the top of the hill and the house, and then finding which chain gives the smallest total time for the journey. The path so selected is absolutely the best and has been selected by what I shall call the "strategy" of the problem. (I shall use the words "strategy" and "tactic" throughout this paper in the particular senses defined, and with no correspondence to other usages, though in some cases our "strategy" will correspond to von Neumann's "Minimax strategy.")

Usually, of course, the traveller would not use so elaborate a method. A common method would be to make the selection in stages. He would first select a point about a hundred feet down to which he could get rapidly; then, arrived there, he would select another point a hundred feet lower still to which a rapid descent was possible, and so on till he reached the house. This method I shall call a simple *tactic*, as contrasted with the previous *strategy*. The tactic differs from the former in that the tactic does not take into account the whole of the situation, but proceeds according to a criterion of optimality that is applied locally, stage by stage.

This example, of the traveller on the hill, is really more general and widely applicable than might at first appear. We can in fact replace the hillside by an abstract space and the expected elementary times t_i (on the elementary areas) by some function $f(P)$ of the points P of the space. At this level of abstraction the problem is then to find a curve C between the initial and terminal points, such that the integral

$$\int_C f(P)dP$$

is a minimum.

To see more clearly how the concept can be applied when the problem is not geometrical, let us consider an example of quite a different type. Suppose a specialist in vocational guidance has to allot n given persons to n given jobs, when he has already tested them and has made predictions (n^2 in number) of the "suitability" (on some scale) of each person for each job. His "goal" is such an assignment as will maximise the suitabilities *over the whole set*. Thus, it might happen that three people A, B, and C had the suitabilities shown in the Table for jobs I, II and III.

	I	II	III
A	9	10	4
B	5	8	4
C	3	2	1

An " assignment " (of people to jobs) is determined when we have selected three entries, no two being in the same row and no two in the same column.

Here the strategy would consist of computing the totals corresponding to each of the *n!* possible assignments. In this example there are $1 \times 2 \times 3$, i.e., 6 possible assignments ; trials soon show that the best is that which gives *A* the job I, *B* the job II, and *C* the job III. This scores 18, and no other assignment scores more.

If *n* is large, the computations become prohibitively heavy ; a possible tactic is then to proceed by first picking out the largest entry in the table, then cancelling out that person and that job, then selecting the next largest in the remaining $n - 1$ persons and jobs, and so on to the completion of the assignment. If it is applied to the table, then the score of 10 first determines that *A* is to be given job II, then the 5 gives *B* the job I and then *C* gets job III.

This tactic scores 16, little less than the maximum of 18. We see therefore that it may be possible to achieve fairly easily a score that is only slightly below the best when the best itself is quite impossible of achievement (as would have been the case in this example had *n* been thirty instead of three).

The same principle occurs in chess-playing. If, toward the end of a game, player *A* can see how to beat *B* in spite of all defences, then *A* will be following what I have called a strategy. Often, however, this perfect way to the goal cannot be perceived ; then *A* looks for some move that will achieve a temporary or local advantage, such as capturing *B*'s Queen, or covering the largest area of the board, or promoting a pawn, etc. Such a move is a tactic.

Strategy and Tactic.

Our next step is to see more clearly what is the relation between these two —to show them as derivatives of a single concept. Let us go back to the man on the hill.

Let us assume that by now he has discovered everything about the hillside, so that he now knows the true minimal time T_i necessary for going from every point P_i of the hillside to the house. This information he has conveniently schematized on his map by drawing a series of *isochronic curves* such that each curve joins all those places that are situated at the same (minimal) time from the house. Thus, the curve marked " 5 minutes " would run through all those places from which the house can be reached in 5 minutes but not less.

Once the map has been prepared, a simple reasoning, borrowed from the calculus of variations, shows that, given any starting point, *the optimal path to the house is one that cuts every isochronic curve at right angles.* (Technically we may express this by saying that the optimal paths are geodesics in a variation problem and that the isochronic curves are the " transversal " of the problem.)

From the practical point of view, the introduction of the isochronic curves is not a mere artifice. In some problems a computation of the solution may be obtained easily if the path is computed backwards from the goal. Sometimes, though more rarely, it happens that computation of the isochronic curves is actually the quickest way to the solution—the most economical in the number of arithmetical operations. Thus, in the problem of assigning jobs we saw that

$n!$ sums involving $(n - 1).n!$ additions had to be performed; if, however, we follow a method derived from the considerations just given, the number will not exceed $n(2^{n-1} - 1)$, which is far fewer; for instance, if n is 12, the number of elementary additions falls from 5,269,017,600 to 24,564. Similar reasoning has been used by Hoffman to design codes of minimal redundancy.

The introduction of isochronic curves enables us now to see more clearly the relation between a strategy and a tactic. For if a point follows a path that is everywhere at right angles to the isochronic curves then it is also moving in such a way as to make maximal the instantaneous decrease of the remaining time—it is moving optimally according to the *local* conditions. In other words, if our man behaved as if acted on by a field of force deriving from a potential (which had the isochronic curves as equi-potential lines), then his path would be identical with that determined by the optimal strategy. It is now clear that once the isochronic curves are given, i.e., *once the map's projection has been changed to that of the really optimal metric, the distinction between strategy and tactic disappears.*

Classification of Imperfect Tactics.

Having considered the optimal path, let us turn next to consider the case of the path that is grossly non-optimal, to that, say of a stone rolling down towards the valley under the action of gravity. According to the laws of physics, the stone, at each moment, is falling in such a direction as takes it the longest distance down in the shortest time, when only immediately local conditions are considered. Thus the difference between the behaviours (or paths) of man and stone is simply a difference between the fields which direct them; the man's field is truly optimal, for it is based on the isochronic curves, and the stone's field is that of its crudest approximation—the Newtonian field of gravitation.

In addition to these two fields are others; what can we say *a priori* of their properties and of their values as tactics?

First they may be classified according to what I shall call their "span of foresight"; thus, if the man coming down the hill plans each next move according to the details of the next hundred feet he will do better than if he were to plan only over the next ten feet. The spans of foresight are here a hundred feet and ten feet respectively.

Should the span of foresight be equal to the distance from the goal, then obviously the tactic and the strategy become identical.

The second characteristic that is to be considered is the behaviour's "flexibility." Suppose the span of foresight is "100 feet below"; once the man has covered half this distance he may discover that his provisional goal was not the best, and that he should now take a different path for a different goal, again at one unit of foresight ahead. Clearly, in the strategy of the traveller with complete foresight the concept of flexibility plays no part, and neither does it at the other extreme—the case of the rolling stone (for the latter's steps are assumed to be infinitesimal, so that no smaller step is possible). It is in the intermediate degrees that the concept becomes important.

After these preliminaries, it will be seen that any goal-seeking behaviour may be classified on a scale of strategies and tactics, each one depending on a general function representing some measure of the distance between the present position and the goal. Thus, for the stone, the function depends on the present altitude above the bottom of the valley. Had the stone been a piece of iron and the goal a big magnet, the function would have depended on the object's present distance from the magnet. Other models would have led to other functions ; so we have many tactics, each depending on a single function. The complete specification, and full classification, are given when to this function we add the span of foresight and the degree of flexibility, the latter being conveniently measured by the minimal time at which the provisional goal may be replaced by another.

The Stochastic Environment.

What we have done so far is to show that the " strategy " is simply one of the tactics : it is that extreme tactic based on the best function as given by the isochronal curves. It is now instructive to show conversely (so close is the relation between them) that any tactic may be viewed as some sort of strategy. This is so if the time t_i taken in crossing each elementary area is not permanently constant, as we have assumed so far, but depends on other factors of which we know only the probability of their values. Thus one of the areas might be a marsh that is easy to cross when the weather has been fine but difficult to cross after rain ; if the traveller does not know what has happened prior to a *particular* trip, he can give only a probability to any particular " duration of crossing." Again, another area might be a lake, to be crossed only by a boat which may or may not be available, and so on. More generally we can assume that the elementary times t_i are not given fixed values but are given fluctuating values, depending on some random or " stochastic " process.

If now we apply the theory of inductive behaviour as defined by Wald on the Ville-von Neumann principles, we find that *the optimal strategy is just the simple tactic of attempting to do one's best on a purely local basis.*

To illustrate this thesis let us consider some well known examples. First consider that of the dog that wants to run to his master, who is himself walking steadily in a definite direction. If the dog is something of a computer it will perform an integration and will go directly to the place at which their relative velocities will enable them to meet. If the dog is not so clever it will continuously run directly towards where its master is at that moment. This second tactic is, of course, the simpler of the two, but is inferior if the master is moving uniformly. If, however, the master is making steps backward and forward in a totally random way, as if he were undergoing a Brownian movement, then this second tactic can be shown, by mathematical proof, to be actually *the best one possible.* The tactic has become the strategy.

Here is a second example. A number of lengths of telephone wire must be joined end to end to complete a long-distance line. Each length imposes some small but characteristic distortion a_i on the message. The overall distortion is the sum of the individual distortions, but by reversing a length before it is joined the distortion can be added positively or negatively. How should the lengths be joined if the total distortion, $\Sigma \pm a_i$ is to be a minimum?

If a hundred lengths are to be connected, the absolute optimum can hardly be achieved, for the number of sums that would have to be calculated (if all possibilities are to be explored) is prohibitively large. What is the engineer to do? In practice, he adds the lengths one by one, at each new addition adding the wire this way or that so as to make the sum as small as possible at that particular stage. This tactic has been found to give quite satisfactory results, its success, perhaps, depending on the fact that this tactic really is the best possible if the wires are provided, and have to be connected, one by one.

Other examples of this thesis—that the optimal strategy often consists of doing one's best on a local basis—is also used in Fano's method for finding an optimal code, and in Gavrilov's method for building up a switching system that will represent a given logical function at minimal cost.

CONCLUSION.

In this paper I have attempted to show something of what is implied by " goal-seeking " when the whole situation is more complex than that occurring in, say, a simple thermostat. It is clear that the further study of these situations will have to be made mathematically. It is also clear that much more development will be necessary on the purely mathematical side, for much of the necessary mathematics will have to be developed specially. To the mathematician many of the problems raised are new, and will need new methods.

In particular we need to know more about how much efficiency is lost when the span of foresight and the degree of flexibility are not optimal. The problem is not made easier by the fact that often the parameters do not enter the problem with random values, so the appropriate theorems will have to be stated in a somewhat unusual form.

The purpose of this paper, however, was not to enter into these technicalities but to show in a general way how the introduction of mathematics into this branch of psychology might itself be a worth-while tactic.

SUMMARY.

When " goal-seeking " behaviour is considered in situations of more than the most elementary type, the problems that arise are related to those of strategies and tactics.

I have attempted to show that clear-cut principles are involved, capable of mathematical treatment.

It appears likely that among the factors of special importance are those of " span of foresight " and " degree of flexibility."

The case has also been considered in which the organism faces an environment that can be characterized only in terms of probability.

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