(T) can also be expressed topologically, using the Fell Topology on the space of all unitary representations of G. In this setting, property (T) amounts to saying that the trivial representation is an isolated point of this space.

Definition 2.2. *Let G be a group with a finite generating set S and let π be an irreducible representation of G with representation space V_π. Define the* Kazhdan constant

$$K_G(S,\pi) := inf_{\xi \in S(V_\pi)} max_{s\in S} \|\pi(s)\xi - \xi\|$$

where $S(V_\pi) = \{\xi \in V_\pi : \|\xi\| = 1\}$.
We define also:

$$K_G(S) := inf\{K_G(S,\pi) : \pi \text{ is irreducible and non trivial}\}$$

We cite here the main result of Bacher and de la Harpe [BD] concerning Kazhdan constants for the symmetric groups:

Theorem 2.3. *For the symmetric group S_n with the Coxeter system*

$$S = \{(1,2),(2,3),...,(n-1,n)\}$$

we have:

$$K_{S_n}(S) = \sqrt{\frac{24}{n^3-n}}.$$

2.2. **Coxeter groups of type B.**

Definition 2.4. *The* Coxeter group *B_n is the group of signed permutations of $\{1,...,n\}$. Namely, B_n consists of all permutations π of $\{-n,...,-1,1,...,n\}$ such that $\pi(-k) = -\pi(k)$ for all $1 \le k \le n$.*

We represent elements of B_n in cycle notation as permutations of $\{-n,...,-1,1,...,n\}$. The elements

$$s_0 = (1,-1)$$

and

$$s_i = (i-1,i)(-(i-1),-i) \quad (1 \le i \le n-1)$$

generate B_n and satisfy the relations:

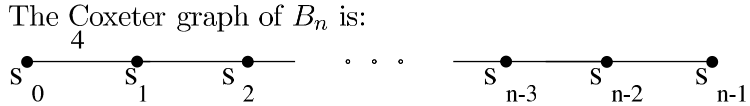
$$s_i^2 = 1 \quad (\forall i)$$

$$s_is_j = s_js_i \quad (|i-j|>1),$$

$$(s_is_{i+1})^3 = 1 \quad (1 \le i \le n-1),$$

$$(s_0s_1)^4 = 1.$$

They are called *Coxeter generators.* Denote $S_{B_n} = \{s_0, s_1, ..., s_{n-1}\}$.
The Coxeter graph of B_n is:



2.2.1. *Representations of the groups of type B.* We start with some definitions:

Definition 2.5. *Let n be an integer. A partition of n of length l is a sequence $\alpha = (a_1, ..., a_l)$ of nonnegative integers such that $a_1 \geq a_1 \geq ... \geq a_l$ and $a_1 + ... + a_l = n$. The size of α is defined by: $|\alpha| = a_1 + ... + a_l (= n)$.*

A partition $\alpha = (a_1, ..., a_l)$ of n can be represented by an array of n boxes in l rows with row i containing a_i boxes, $(1 \leq i \leq l)$. This is called the *Young diagram* of the partition α.

Definition 2.6. *A* double partition, *$\lambda = (\lambda_1, \lambda_2)$ of size n is an ordered pair of partitions λ_1 and λ_2 such that $|\lambda_1| + |\lambda_2| = n$.*

Every double partition $\lambda = (\lambda^1, \lambda^2)$ of size n is equipped with a *double Young diagram* which is a pair of Young tableaux, one for each λ^i. We also use the term *shape* for a double Young diagram.

The irreducible representations of the groups B_n are indexed by the shapes $\lambda = (\lambda^1, \lambda^2)$ such that $|\lambda| = |\lambda^1| + |\lambda^2| = n$.

A standard Young tableau $L = (L^{\lambda^1}, L^{\lambda^2})$ is a filling of the Young diagram of λ with the numbers $1, ..., n$ such that in each of L^{λ^1} and L^{λ^2} separately, numbers are increasing along rows and along columns. For example,

1	3	4	9
6	8		

2	5	7

is a standard tableau of the shape $((4, 2), (3))$

2.3. **The groups of type D.** The Coxeter group D_n is the group of signed permutations of $\{1, 2, ..., n\}$ with an even number of negative signs. More precisely, D_n consists of all permutations π of $\{-n, ..., -1, 1, ..., n\}$ such that $\pi(-k) = -\pi(k)$ for all $1 \leq k \leq n$ and an even number of the numbers of $\pi(1), \pi(2), ..., \pi(n)$ are negative. We represent elements of D_n in cycle notation as permutations of $\{-n, ..., -1, 1, ..., n\}$. The element

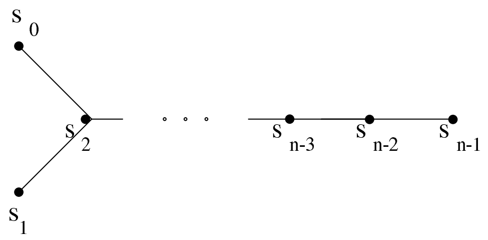
$$s_0 = (1, -2)(2, -1)$$

together with $s_i = (i-1, i)(-(i-1), -i), (\quad 1 \leq i \leq n-1)$ generate D_n and satisfy the relations:

$$s_i s_j = s_j s_i, \quad (|i-j| > 1, \quad i, j > 0),$$
$$s_0 s_j = s_j s_0 \quad (j \neq 2)$$
$$s_0 {s_2}^3 = 1,$$
$$s_i {s_{i+1}}^3 = 1, \quad (1 \leq i \leq n-1),$$
$$s_i^2 = 1 \quad (1 \leq i \leq n-1).$$

The Coxeter group D_n can be realized as a normal subgroup of the Coxeter group B_n of index 2.

The Coxeter graph of The groups of type D is:



2.3.1. *Representations of* D_n. Being a normal subgroup of B_n of index 2, D_n essentialy inherits its irreducible representations from B_n. By Clifford theory, the restriction to D_n of an irreducible representation corresponding to a shape $\lambda = (\lambda_1, \lambda_2)$ with $(\lambda_1 \neq \lambda_2)$ is an irreducible representation of D_n. On the other hand, if $\lambda = (\lambda_1, \lambda_2)$ with $\lambda_1 = \lambda_2$ then the restriction to D_n of the corresponding B_n-representation splits into a sum of two non-isomorphic irreducible representations of D_n. All irreducible representations of D_n are obtained in this fasion.

2.4. **The groups of type G(r,n).** Let $G(r,n) = C_r \wr S_n$ be the wreath product of C_r and S_n . $G(r,n)$ is a unitary reflection group consisting of all monomial matrices (i.e., products of diagonal and permutation matrices) of order $n \times n$ whose non-zero entries are complex r-th roots of unity.

Abstractly, the group $G(r,n)$ can be presented by a set of generators $S_W = \{s_0, s_1, ..., s_{n-1}\}$ with the following set of relations:

$$s_0^r = 1$$
$$s_i^2 = 1 \quad (i = 1, ..., n-1)$$
$$s_0 s_1 s_0 s_1 = s_1 s_0 s_1 s_0$$
$$s_i s_j = s_j s_i, (|i-j| \geq 2)$$
$$s_i s_{i+1} s_i = s_{i+1} s_i s_{i+1} \quad (i = 1, ..., n-1)$$

Note that for $r = 2$, $G(r,n) = B_n$.

2.4.1. *Representations of G(r,n).* The representation theory of the groups $G(r,n)$ is a generelization of the representation theory of B_n. The only difference is that one works with $n-$ tuples of partitions and Young tableaux instead of double partitions and double Young tableaux. For more details see e.g. [HR].

3. Exact Kazhdan constants

Our first main result is an exact computation of the Kazhdan constant for the Coxeter groups of type B.

Theorem 3.1. *The Kazhdan constant of the group* B_n *with respect to the set of Coxeter generators* S_{B_n} *is:*

$$K_{B_n}(S_{B_n}) = \sqrt{\frac{4}{\sum_{j=1}^{n} \left(1 + \sqrt{2}(j-1)\right)^2}}$$

Sketch of the proof: The Kazhdan constant is achieved by the following procedure. We begin by computing an upper bound for the Kazhdan constant of a specific representation of the group B_n, namely the natural representation which reflects the action of the Coxeter generators on the Euclidean space $\mathbb{R}^n$. The vector of $\mathbb{R}^n$ on which the upper bound is attained is moved the same amount by all of the Coxeter generators and thus taken from the central chamber of the natural action of B_n on $\mathbb{R}^n$. This vector can be computed by solving some linear equations.

The next step is to prove that that the value for the natural representation serves also as a lower bound for every nontrivial irreducibe representation of B_n. Since the natural representation of B_n is irreducible, we conclude that this value is the Kazhdan constant for the whole set of representations of B_n.

The proof that the upper bound is also a lower bound is combinatorial in nature. The idea is to divide the representation space of every irreducible representation which is composed of standard Young tableaux of some given shape into n subspaces chosen according to the digit located in top left box of the second tableau. The subspaces are chosen in such a way that almost all of the Coxeter generators act invariantly on every single subspace.

We note that that a similar idea appears first in [BD], but their choice of the box which splits the representation space into subspaces was inadequate to our case. Moreover, due to the structure of the representation theory of B_n, our choice yields a more elegant proof.

We deal next with the family of Coxeter groups of type D. Here, since some of the irreducible representations of D_n split into two irreducible representations the computation is much more complicated. The case of non-splitting representations is very similar to the case of the groups of type B while the case of splitting representations requires a new parameterization of the basis by tableaux. The Kazhdan constant for the Coxeter groups of type D is given in the following:

Theorem 3.2. *The Kazhdan constant of the Coxeter groups of type D with respect to the set S_{D_n} of Coxeter generators is:*

$$K_{D_n}(S_{D_n}) = \sqrt{\frac{2}{\sum_{j=2}^{n}(j-1)^2}} = 2\sqrt{3}\sqrt{\frac{1}{n(2n^2-3n+1)}}$$

The following result generalizes theorem 3.1

Theorem 3.3. *The Kazhdan constants for the groups $G(r,n)$ with respect to the set S_W of generators is:*

$$K_{G(r,n)}(S_W) = \sqrt{\frac{|\rho_r - 1|^2}{\sum_{j=1}^{n}\left(1 + \frac{|\rho_r-1|}{\sqrt{2}}(j-1)\right)^2}}.$$

where $\rho_r = e^{\frac{2\pi i}{r}}$.

Naturally, the next family of groups whose Kazhdan constant is interesting is the complex reflection groups $G(r,n,p)$. Work in this direction is in progress.

Acknowledgments

This work is a part of a Ph.D. thesis at Bar-Ilan University carried out under the supervision of Ron Adin and Yuval Roichman. The author would like to thank Alex Lubotzki, Pierre de la Harpe and Andre Zuk for useful discussions.

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