THE LOST NOTEBOOK AND PARTIAL THETA FUNCTIONS

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In the Lost Notebook, Ramanujan wrote many amazing identities for partial theta functions, such as

$$1+\frac{x^2(1-x)}{(1+ax^2)(1+\frac{x^2}{a})}+\frac{x^4(1-x)(1-x^3)}{(1+ax^2)(1+ax^4)(1+\frac{x^2}{a})(1+\frac{x^4}{a})}+\cdots$$
$$=(1+a)(1-ax+a^2x^3-a^3x^6+\cdots)$$
$$-a(1-x)(1-x^3)(1-x^5)\cdots\frac{1-ax+a^3x^5-a^4x^8+\cdots}{(1+ax^2)(1+ax^4)\cdots(1+\frac{x^2}{a})(1+\frac{x^4}{a})}$$

and

$$1+\frac{q(1-q)}{(1+aq)(1+\frac{q}{a})}+\frac{q^2(1-q)(1-q^3)}{(1+aq)(1+aq^2)(1+\frac{q}{a})(1+\frac{q^2}{a})}$$
$$=(1+a)(1-aq+a^2q^3-a^3q^6+\cdots)$$
$$-a\cdot\frac{(1-q)(1-q^3)\cdots(1-a^2q^2+a^4q^6-\cdots)}{(1+aq)(1+aq^2)\cdots(1+\frac{q}{a})(1+\frac{q^2}{a})}$$

In this talk I will try to explain the origin of these identities, and will show that most partial-theta formulae from the Lost Notebook can be embedded in infinite hierarchies of such identities. This will reveal an unexpected connection between partial theta functions and Rogers–Ramanujan-type identities.

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