Counting Braids and Laminations

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École des Mines de Paris & Université Paris Diderot (LIAFA)

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Braid Groups

What are braids?

1. Intertwined strands

 

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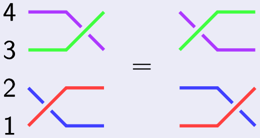
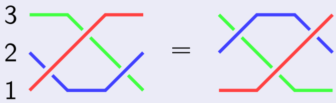
2. Isotopy group of braid diagrams





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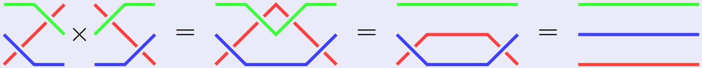

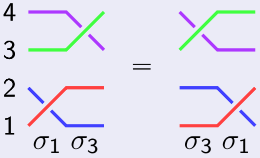

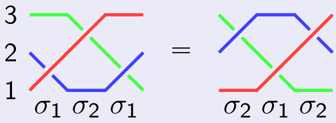

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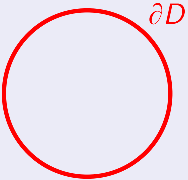

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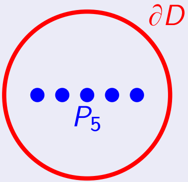

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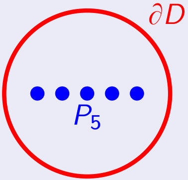
3. Isotopy group of homeomorphisms of $\mathbb{C}$ **that fix ∂D pointwise** and **let P_n globally invariant**: $\mathcal{B}_n = \frac{\mathrm{Hom}(\mathbb{C}, P_n \leftrightarrow P_n, \mathbf{Id}_{\partial D})}{\mathrm{Hom}_0(\mathbb{C}, P_n \leftrightarrow P_n, \mathbf{Id}_{\partial D})}$.



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   $\mathcal{B}_n = \langle \sigma_1, \ldots, \sigma_{n-1} \mid \sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1}, \sigma_i \sigma_j = \sigma_j \sigma_i \text{ if } \geqslant i+2 \rangle$.
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Coxeter Group:

$\mathfrak{S}_n = \langle \sigma_1, \ldots, \sigma_{n-1} \mid \sigma_i^2 = 1, \sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1}, \sigma_i \sigma_j = \sigma_j \sigma_i \text{ si } j \geqslant i+2 \rangle$.

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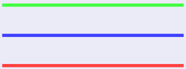

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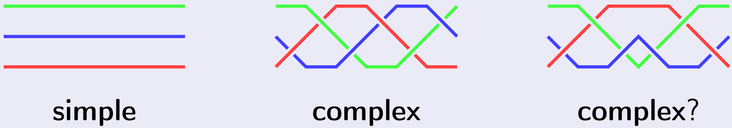

- $\|\alpha\|$ = minimal number of crossings
- $\|\alpha\|$ = distance to ε in a Cayley graph: $\|\alpha \cdot \beta\| \leqslant \|\alpha\| + \|\beta\|$

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- Computing $\|\alpha\|$: very hard (easy up to a multiplicative factor $n!$)
- Computing $N^{(k)} = \#\{\alpha : \|\alpha\| = k\}$: seems very hard

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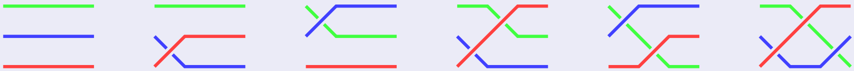

generated by simple braids

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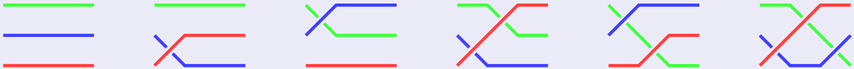
Idea #2: distance to ε in **another** Cayley graph



generated by **simple** braids

- $\|\alpha\|_2$ = distance to ε in a Cayley graph: $\|\alpha \cdot \beta\|_2 \leqslant \|\alpha\|_2 + \|\beta\|_2$

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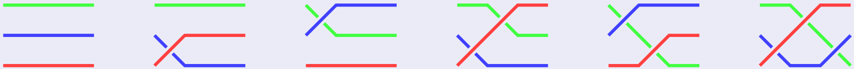
Idea #2: distance to ε in **another** Cayley graph



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- $\|\alpha\|_2 =$ distance to ε in a Cayley graph: $\|\alpha \cdot \beta\|_2 \leqslant \|\alpha\|_2 + \|\beta\|_2$
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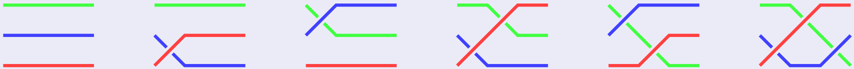

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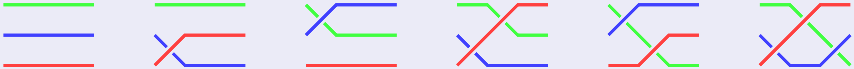

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- Computing $\|\alpha\|_2$: easy
- Computing $N_2^{(k)} = \#\{\alpha : \|\alpha\|_2 = k\}$: easy ($\sum_{k \geqslant 0} N_2^{(k)} z^k$ is rational)

Contents

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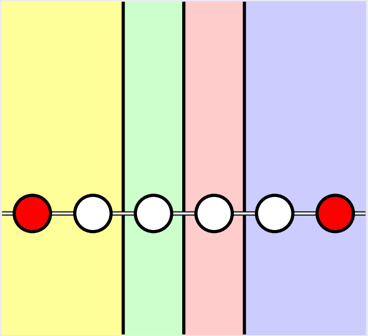
Trivial band lamination:



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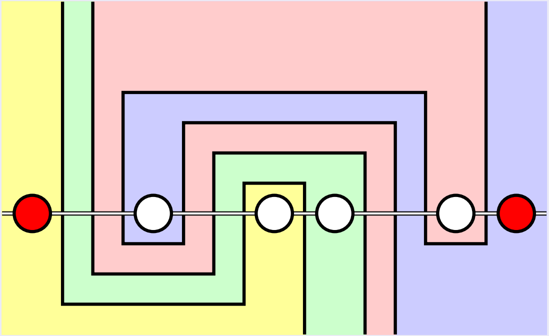

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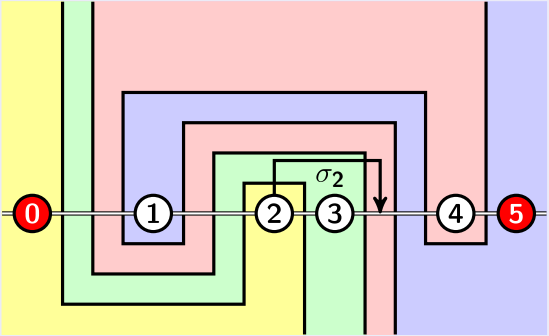

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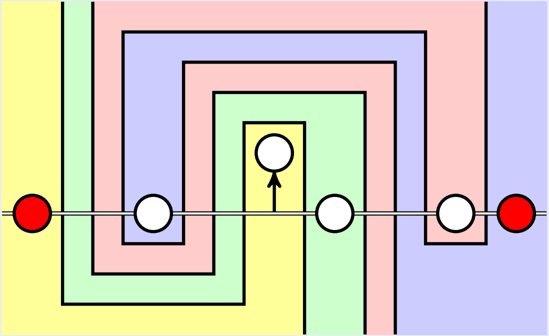

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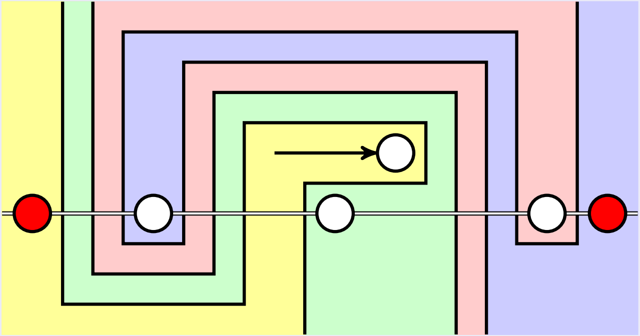

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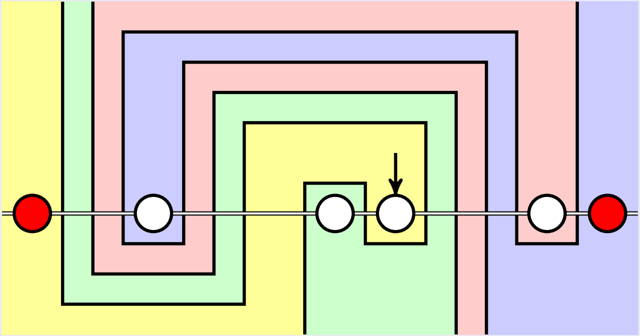

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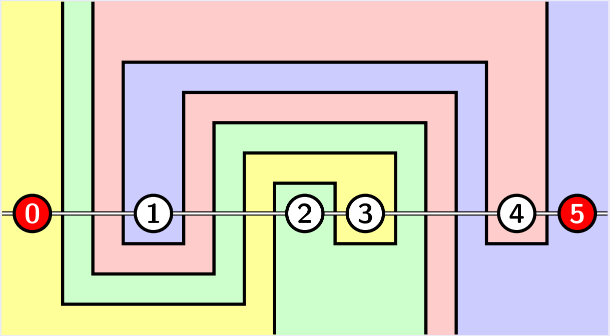

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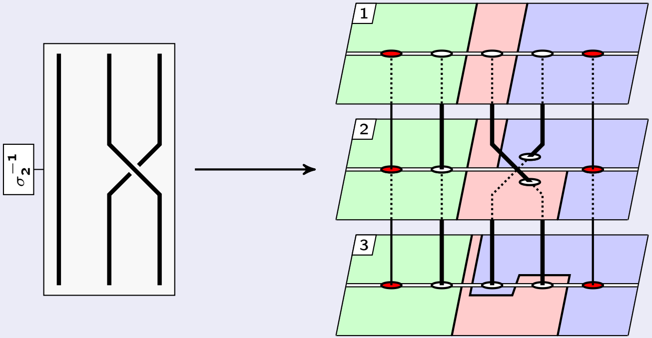

Braid Acting on a Band Lamination

Braid $\equiv$ Band lamination

$\mathcal{B}_n$ acts **faithfully** and **transitively** on $\mathcal{L}_n^b$:

$\mathcal{B}_n = \{n\text{-strand braids}\}$
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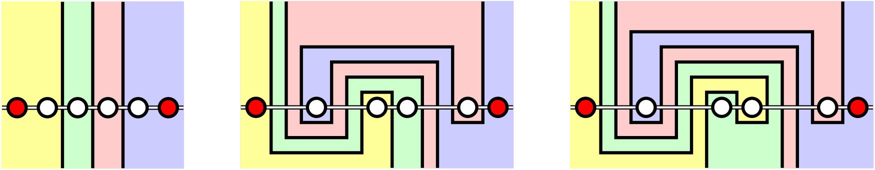

ε $\quad$ $\sigma_1^{-1}\sigma_2^{-1}\sigma_3$ $\quad$ $\sigma_2\sigma_1^{-1}\sigma_2^{-1}\sigma_3$

Laminations and Complexity

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Idea #3: a band lamination whose arcs often cross $\mathbb{R}$

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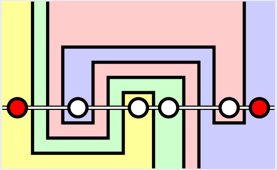

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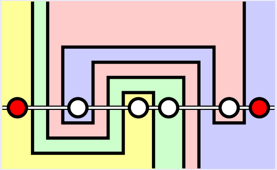

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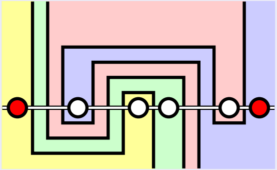

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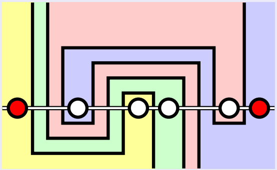

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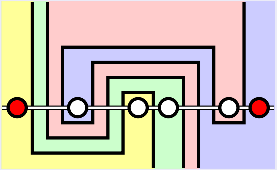

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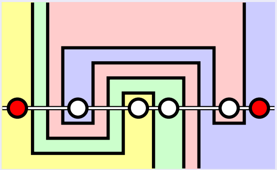

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- Computing $N_3^{(k)} = \#\{\alpha : \|\alpha\|_3 = k\}$: not obvious...

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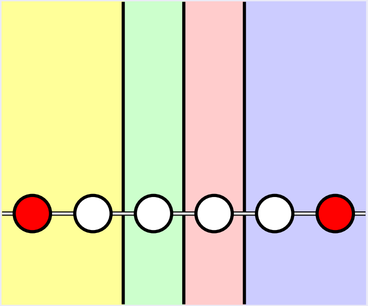

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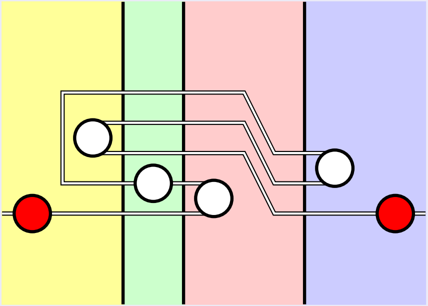

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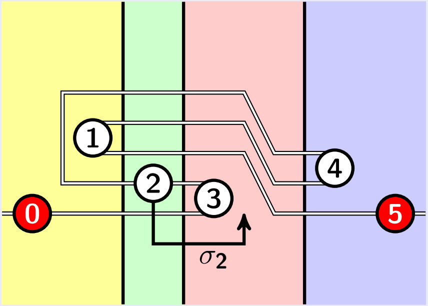

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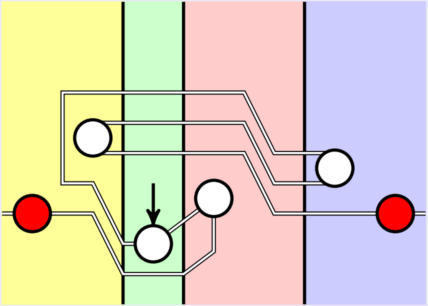

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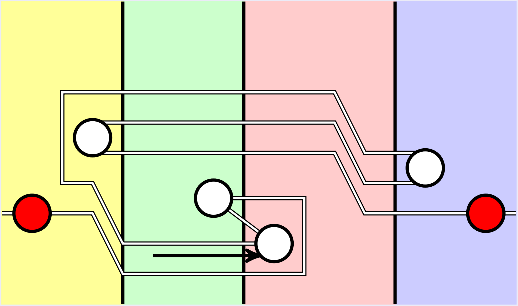

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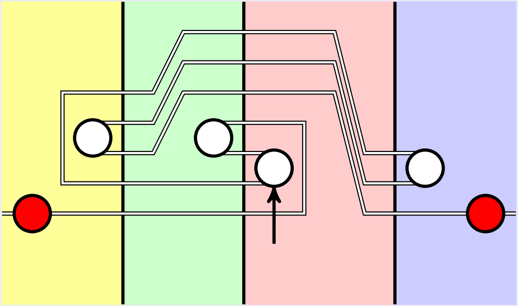

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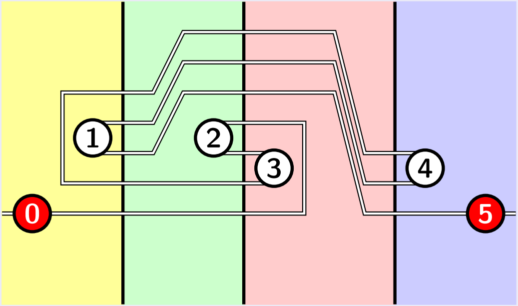

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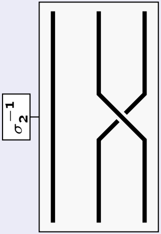

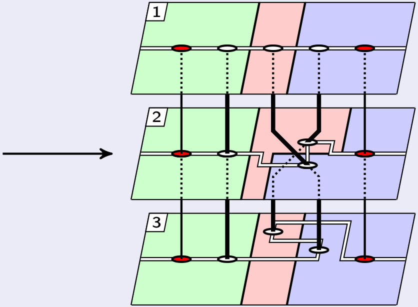

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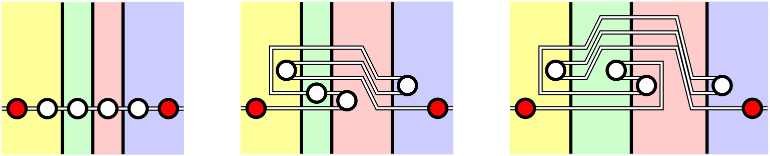

ε | $\sigma_1^{-1}\sigma_2^{-1}\sigma_3$ | $\sigma_2\sigma_1^{-1}\sigma_2^{-1}\sigma_3$

Laminations and Complexity

What is a complex braid?

Idea #4: a lamination whose ray often crosses $\mathbf{L}^b_\varepsilon$

Laminations and Complexity

What is a complex braid?

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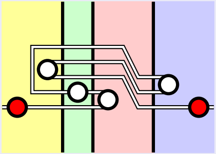

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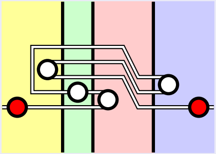

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- $\|\alpha\|_4 =$ cardinality of $\alpha(\mathbf{L}_\varepsilon^r) \cap \mathbf{L}_\varepsilon^b$

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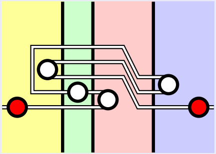

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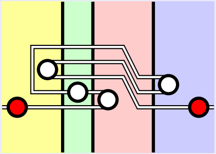

Complex braid

- $\|\alpha\|_4$ = cardinality of $\alpha(\mathbf{L}_\varepsilon^r) \cap \mathbf{L}_\varepsilon^b = \|\alpha^{-1}\|_3$
- Computing $N_4^{(k)} = \#\{\alpha : \|\alpha\|_4 = k\} = N_3^{(k)}$: **not so hard**...

Laminations and Complexity

Why do we have $\|\alpha\|_4 = \|\alpha^{-1}\|_3$?

Pull α's ray tight!

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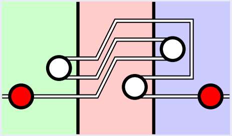

$|\sigma_2\sigma_1^{-1}(\mathbf{L}_\varepsilon^r) \cap \mathbf{L}_\varepsilon^b|$

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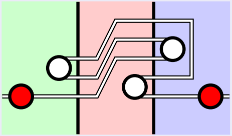

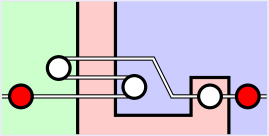

$$|\sigma_2\sigma_1^{-1}(\mathbf{L}^r_\varepsilon) \cap \mathbf{L}^b_\varepsilon| \quad = \quad |\sigma_2^{-1}(\mathbf{L}^b_\varepsilon) \cap \sigma_1^{-1}(\mathbf{L}^r_\varepsilon)|$$

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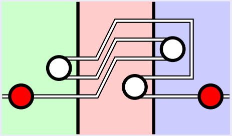

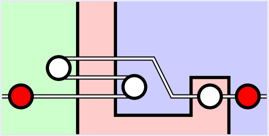

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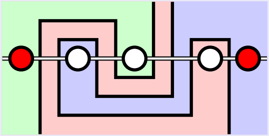
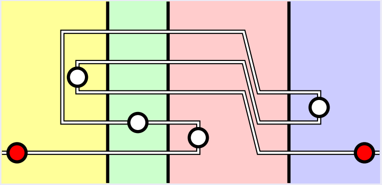


$$= |\sigma_1\sigma_2^{-1}(\mathbf{L}^b_\varepsilon) \cap \mathbf{L}^r_\varepsilon|$$

Counting Laminations

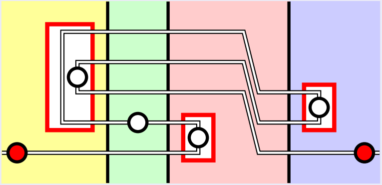
How can we count (radial) laminations?



Counting Laminations

How can we count (radial) laminations?

1. Identify **mirrors**



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How can we count (radial) laminations?

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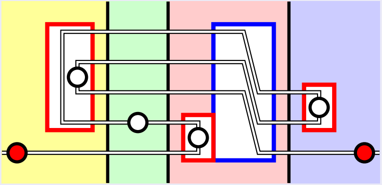
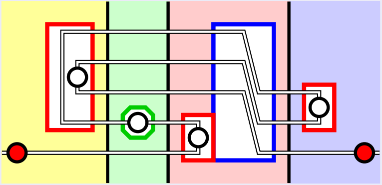


Counting Laminations

How can we count (radial) laminations?

1. Identify mirrors and their periscopes and transparent holes



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How can we count (radial) laminations?

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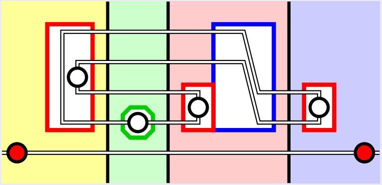

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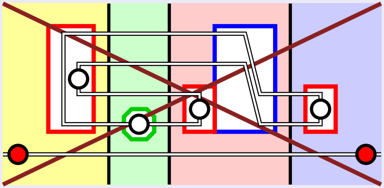

Counting laminations: 1 or 2 strands

1-strand braids:

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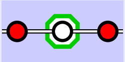

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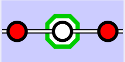
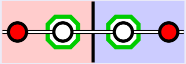


2-strand braids: $N_4^{(k)} = \mathbf{1}_{k=1}$



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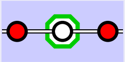

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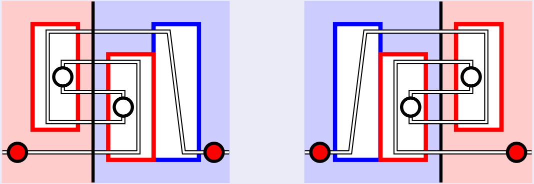

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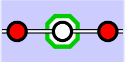

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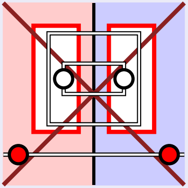

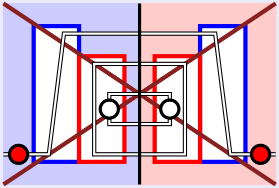

Counting laminations: 3 strands

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$$N_4^{(k)} = \mathbf{1}_{k=2} + 2\varphi(k/2+1) \cdot \mathbf{1}_{k \in 2\mathbb{N}+4}$$

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$$N_4^{(k)} = \mathbf{1}_{k=2} + 2\varphi(k/2+1) \cdot \mathbf{1}_{k \in 2\mathbb{N}+4} - 2 \cdot \mathbf{1}_{k \in 4\mathbb{N}+6}$$

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$$
\begin{aligned}
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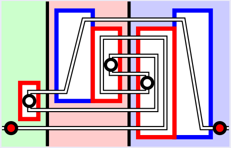
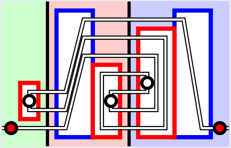
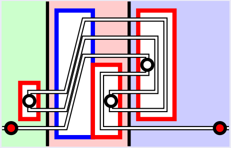
Typical cases:







Counting laminations: 4 strands or more

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Conjecture

$M_\ell = \Theta(\ell^{2n-4})$

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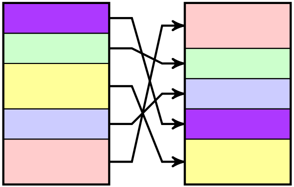
Is this permutation cyclic?



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Conclusion

Next goals

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Thank you!