Dimension groups and dynamical systems
Errata and complements

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1 Chapter 1

page 7, (Eduardo Scarparo, april 2022). replace Example 1.1.1 by: As a simple example, consider $X = [0, 1]$, which is metric and compact as a closed interval of the real line $\mathbb{R}$. The transformation $T : x \mapsto x^2$ is a continuous map from $X$ onto $X$.

page 8, replace Example 1.1.2 by: Given $\alpha \in \mathbb{R}$, the transformation $T : x \mapsto x + \alpha \mod 1$ is not continuous at $x = 1 - \alpha \mod 1$ and thus the pair $([0, 1], T)$ is not a topological dynamical system. If we consider, instead of $[0, 1]$, the torus $T = \mathbb{R}/\mathbb{Z}$ in which 0 and 1 are identified, the transformation $T$ is simply the translation $T_\alpha : x \mapsto x + \alpha$ and becomes a homeomorphism on $T$. The system $(T, T_\alpha)$ is called the rotation of angle $\alpha$.

page 9, Proposition 1.1.3. ...for a topological dynamical system $(X,Y)$ with $T$ surjective.

page 12, line 15. $f : X \to \mathbb{N}_+$. 

page 15, line -8 (Simon Binder, january 2023). $F_{n+2} = F_{n+1} + F_n$

page 16, line -8. Two measure-theoretic systems $(X,T,\mu)$ and $(X',T',\mu')$...

page 16, line -6. such that $\varphi \circ T(x) = \varphi \circ T'(x)$ for every $x \in X_1$ and $\mu(U) = \mu'(\varphi(U))$ for every Borel subset $U \subset X_1$.

page 24, line -5. A nonempty shift space is recurrent if and only if it is irreducible.

page 25, line 10. A nonempty shift space $X$ is uniformly recurrent if ...

page 25, line -15 (Simon Binder). Then $S^t x \in [u]_X$.

page 38, line -12 (Simon Binder). $M = M(\varphi)$.

page 39, line 12. Let $M$ be the incidence matrix of a primitive morphism $\varphi$ with dominant...

page 39, line 19. results directly from (B.10)

page 40, line 17. Every return word to $w$ is then a factor of the image by $\sigma^n$ of a word of length at most equal to the maximal length of the return word to words of length 2.

page 40, line -8. the maximal length $R$ of return words to words of length 2 is 8.

page 43, line 2. $(X^c,T)$ is isomorphic to ...
2 Chapter 2

Let $r, s > 0$ be integers such that $p(g) \geq r/s$ ...

page 101, line -19. $\alpha: (\mathbb{Z}_n, \mathbb{Z}_n^+ \to (G, G^+)$

page 101, line -15. $(G, G^+) = (\mathbb{Z}_k^p, \mathbb{Z}_k^p)$. Change $n$ to $p$ (5 times).

3 Chapter 3

Add: By convenience, we prove the result for a Cantor minimal system, although it holds more generally for minimal dynamical systems (see the two alternative proofs given in Exercises 3.10 and 3.11).

page 115, line -8 Let $(X, T)$ be a minimal Cantor system...

page 120, line 7, replace $uA^N = uxA^N$ by $[u] = [ux]$.  

page 136, line 4. $\mu_2(W) = \frac{1}{\mu(W)}\mu(W \cap V)$

page 138, line 8. We shall see later a different proof using dimension groups and also that the result is true more generally for minimal substitution shifts.

page 145, line 16 (Christian Choffrut, March 2023). the map $(x, y) \mapsto x\chi[0] + y\chi[1]$.

page 155, line -7 (Marie-Pierre Béal, March 2022). A direct proof of the fact that two Sturmian shifts of slopes $\alpha, \beta$ are conjugate if and only if $\alpha = \beta$ or $\alpha = 1 - \beta$ can be found in [1] (see also [2, Theorem 5.19] where a proof using eigenvalues is given).

4 Chapter 4

page 159, line 6. Indeed,

page 163, line -13 ((Christian Choffrut, March 2023). is the restriction to $G(\mathfrak{P})$ of the map $R_{B(\mathfrak{P})}$ from $C(B(\mathfrak{P}), \mathbb{Z})$ to $C(X, \mathbb{Z})$.

page 164, line 4 (Simon Binder, January 2022). $\ker I(\mathfrak{P}) \subset \ker I(\mathfrak{P}')$.

page 181, line -1 (Christian Choffrut, March 2023).

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$
5 Chapter 5

page 214, line 3. \( C_1 \supset C_2 \supset \cdots \supset C_n \supset \cdots \)

page 214, line 9 (Simon Binder, Jan 2023). \( j_{n-1} + \ldots + j_1 \)

page 216, line 17 (Simon Binder, Jan 2023). \( \sum_{1 \leq n \leq n_0} j_n = j_{n_0} + \sum_{1 \leq n \leq n_0 - 1} (h_{t_n}(n) - h_{t_n-1}(n-1)) = \ldots \)

page 219, line -1. Note that the dimension group of a stationary properly ordered BV system \((X_E, V_E)\) is the direct limit of a stationary system defined by a primitive matrix. By Theorem 2.5.1, it has a unique state and thus \((X_E, V_E)\) is uniquely ergodic.

page 222 line 9. Add: It is not obvious at all that the strong orbit equivalence is actually an equivalence relation between dynamical systems. This will result from Theorem 6.5.1.

page 238, add Exercise: A pointed conjugacy from \((X,T,x)\) to \((X',T',x')\) is a conjugacy \(\phi\) from \((X,T)\) to \((X',T')\) such that \(\phi(x) = x'\). Let \((V,E,\leq)\) and \((V',E',\leq')\) be two properly ordered Bratelli diagrams. Show that \((V,E,\leq)\) and \((X',E',\leq')\) have a common intertwining if and only if there is a pointed conjugacy from \((X_E, T_E, x_{\min})\) to \((X'_E, T'_E, x'_{\min})\).

Solution: If \((V',E',\leq')\) is obtained from \((V,E,\leq)\) by telescoping, then the corresponding map \(\phi: X_E \rightarrow X'_E\) is clearly a conjugacy such that \(\phi(x_{m,n}) = x'_{m,n}\).
6 Chapter 6

page 245, line 12, are the eventually constant sequences.

page 247, Figure 6.3 (Simon Binder, jan. 2923). ..of \((\mathbb{Z}_{(p_n)}, T)\).

page 252, line -5. Note that this implies that every minimal substitution shift is uniquely ergodic (see Section 5.3.4).

page 253, change Proposition 6.2.2 into Let \(B = (V, E, \leq)\) be a stationary Bratteli diagram. The morphism read on \((V, E, \leq)\) is primitive and eventually proper if and only if the diagram is properly ordered.

Add to the proof, at the beginning: Assume first that \(B\) is properly ordered.

After the end: Conversely, assume that the morphism \(\sigma: A^* \rightarrow A^*\) read on \(B\) is primitive and eventually proper. Since \(\sigma\) is primitive, there is \(n \geq 1\) such that \(|\sigma^n(a)|_b > 0\) for every \(a, b \in A\). Thus the Bratteli diagram \((V, E)\) is simple. Next, since \(\sigma^n\) is left proper for some \(n \geq 1\), there is a unique minimal path using all vertices \((nk, i^n(a))\) for \(k \geq 0\). Similarly, there is a unique maximal path. Thus \((V, E, \leq)\) is properly ordered.

Add after the proof: Note that when \(B = (V, E, \leq)\) is a stationary properly ordered Bratteli diagram, the point \(x_{\min}\) of the BV-system \((X_E, T_E)\) is the unique fixed point of the morphism read on \(B\).

page 260, line -4 (Marie-Pierre Béal, december 2022). Indeed, \(r(\phi \circ r^n(b)) \ell = r(\sigma^n \circ \phi(b)) \ell\) begins with \(\sigma^n(\ell)\) (because \(r\phi(b)\ell\) begins with \(r\ell\)).

page 293, line 11 (Marie-Pierre Béal, march 2022). the word \(\varphi_u(j)u\) appears ....

page 293, line -2 (Marie-Pierre Béal, march 2022). we have \(\varphi_u \circ \sigma^u = \ldots\)

page 295, line 2. Transfer ‘Since \(y\) is not periodic’ to the beginning of the next sentence.

page 296, line 9.

\[ S^{[n]}(x) = \ldots \]

idem line 12 (twice).

page 302, Exercise 6.32. Show that for every sequence \(x = x_0x_1 \cdots \in A^\mathbb{N}\), there are...such that \(x = \lim \phi \circ \sigma_{x_0} \circ \cdots \circ \sigma_{x_n}(\#)\) and thus...

7 Chapter 8

page 367, line 5. \(f(t) = 0\) otherwise.

8 Chapter 9

page 412, line 3. \(k \geq 0\) and ...

page 413, line 6 (Christian Choffrut, march 2023).

\[ n_i = \sum_{j=1}^{t} a_{ij}m_j \]

page 414, line -7. Consequently every isomorphism of \(C^*\)-algebras is an isometry.
are upper triangular with...

\[ q_{k+1} = a_{k+1}q_k + q_{k-1}. \]

since \( R \) is

Add: Let \( \alpha = [a_0; a_1, a_2, \ldots] \) be the continued fraction expansion of \( \alpha \) and let \( p_n, q_n \) be the corresponding sequence of partial quotients (see the definition of a Sturmian algebra).

Let \( \mathfrak{A} = \lim \mathfrak{A}_m \) and \( \mathfrak{B} = \lim \mathfrak{B}_m \)

\[ x^n + x^{n+1} \]

\[ + (a_{n-1} + a_n) x^n + x^{n+1} \]

9 Appendix A

Since \( T \) is onto, if the positive orbit of \( x \) is dense, the positive orbit of \( Tx \) is also dense. Indeed, since the positive orbit of \( x \) contains points arbitrary close to \( y \), the positive orbit of \( Tx \) contains arbitrary close to \( x \). Thus, if \( U, V \) are nonempty open sets, let \( n \geq 0 \) be such that \( T^n x \in V \). Since \( T^n x \) has dense positive orbit, there an \( m \) such that \( T^{n+m} x \in V \) and thus \( U \cap T^m V \neq \emptyset \).

with positive cone \( \mathbb{Z}_+ \times \mathbb{Z} \) and unit \((3, -1)\), which can be normalized to \((1, 1)\) through the automorphism \((\alpha, \beta) \mapsto (\alpha/3, -\beta)\).

with right eigenvectors \( u = [1, 1]^t \) and \( v = [1, -2]^t \).

Define \( \sigma_a \) by

\[ \sigma_a(b) = \begin{cases} \#a & \text{if } b = \# \\ b & \text{otherwise} \end{cases} \]

and set \( \phi(a) = a \) for every \( a \in A \) and \( \phi(\#) = \varepsilon \).

10 Appendix B

F should be \( A \) (twice).

...and, if \( M \) is irreducible, \( \lambda_M \) is the only eigenvalue with a nonnegative eigenvector.

11 Appendix C

with positive cone \( \mathbb{Z}_+ \times \mathbb{Z} \)
References
