Dimension groups and dynamical systems
Errata and complements

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1 Chapter 1

page 7, (Eduardo Scarparo, april 2022). replace Example 1.1.1 by: As a simple example, consider $X = [0,1]$, which is metric and compact as a closed interval of the real line $\mathbb{R}$. The transformation $T\colon x \mapsto x^2$ is a continuous map from X onto X.
page 8, replace Example 1.1.2 by : Given $\alpha \in \mathbb{R}$, the transformation $T\colon x \mapsto x + \alpha \bmod 1$ is not continuous at $x = 1 - \alpha \bmod 1$ and thus the pair $([0,1], T)$ is not a topological dynamical system. If we consider, instead of $[0,1]$, the *torus* $\mathbb{T} = \mathbb{R}/\mathbb{Z}$ in which 0 and 1 are identified, the transformation T is simply the translation $T_\alpha\colon x \mapsto x + \alpha$ and becomes a homeomorphism on $\mathbb{T}$. The system $(\mathbb{T}, T_\alpha)$ is called the *rotation of angle* α.
page 9, Proposition 1.1.3. ...for a topological dynamical system (X,Y) with T surjective.
page 12, line 15. $f\colon X \to \mathbb{N}_+$.
page 15, line -8 (Simon Binder, january 2023). $F_{n+2} = F_{n+1} + F_n$
page 16, line -8. Two measure-theoretic systems (X,T,μ) and (X', T', μ')...
page 16, line -6. such that $\varphi \circ T(x) = \varphi \circ T'(x)$ for every $x \in X_1$ and $\mu(U) = \mu'(\varphi(U))$ for every Borel subset $U \subset X_1$.
page 24, line -5. A nonempty shift space is recurrent if and only if it is irreducible.
page 25, line 10. A nonempty shift space X is *uniformly recurrent* if ...
page 25, line -15 (Simon Binder). Then $S^i x \in [u]_X$.
page 38, line -12 (Simon Binder). $M = M(\varphi)$.
page 39, line 12. *Let* M *be the incidence matrix of a primitive morphism* φ *with dominant...*
page 39, line 19. results directly from (B.10)
page 40, line 17. Every return word to w is then a factor of the image by σ^n of a word of length at most equal to the maximal length of the return word to words of length 2..
page 40, line -8. the maximal length R of return words to words of length 2 is 8.
page 43, line 2. (X^φ, T) is isomorphic to ...

2 Chapter 2

page 89, line -10 (Christian Choffrut, january 2023) Let $r, s > 0$ be integers such that $p(g) \geq r/s$ ...
page 93, line 15 (Simon Binder, january 2023). $i_n(G_n^+)$.
page 96, line 7 (Christian Choffrut, january 2023). if $z \cdot v$ is positive...
page 101, line -19. $\alpha \colon (\mathbb{Z}^n, \mathbb{Z}_+^n) \to (G, G_+)$
page 101, line -15. $(G, G^+) = (\mathbb{Z}^{k_p}, \mathbb{Z}_+^{k_p})$. Change n to p (5 times).

3 Chapter 3

page 115, line -9 (Aurélien Fourré, december 2022). Add: By convenience, we prove the result for a Cantor minimal system, although it holds more generally for minimal dynamical systems (see the two alternative proofs given in Exercises 3.10 and 3.11).
page 115, line -8 Let (X, T) be a minimal Cantor system...
page 116, line -9 (Aurélien Fourré, december 2022). such that $T^n x$ is in U_ϵ.
page 120, line 7, replace $uA^{\mathbb{N}} = uxA^{\mathbb{N}}$ by $[u] = [ux]$.
page 128, line -2. The induced system (U, T_U) is ...
page 131, line 22. a unique Borel probability measure on X^+ satisfying...
page 132, line -8. such that $\mu(T^n[u]) = \pi(u)$ for every $n \in \mathbb{Z}$.
page 136, line 4. $\mu_2(W) = \frac{1}{\mu(V)}\mu(W \cap V)$
page 136, line -7. a Borel set G_j ...
page 138, line 8. We shall see later a different proof using dimension groups and also that the result is true more generally for minimal substitution shifts.
page 145, line 16 (Christian Choffrut, february 2023). the map $(x, y) \mapsto x\chi_{[0]} + y\chi_{[1]}$.
page 155, line -7 (Marie-Pierre Béal, march 2022). A direct proof of the fact that two Sturmian shifts of slopes α, β are conjugate if and only if $\alpha = \beta$ or $\alpha = 1 - \beta$ can be found in [1] (see also [2, Theorem 5.19] where a proof using eigenvalues is given).

4 Chapter 4

page 159, line 6. Indeed,
page 163, line -13 ((Christian Choffrut, March 2023). is the restriction to $G(\mathfrak{P})$ of the map $R_{B(\mathfrak{P})}$ from $C(B(\mathfrak{P}), \mathbb{Z})$ to $C(X, \mathbb{Z})$.
page 164, line 4 (Simon Binder, january 2022). $\ker I(\mathfrak{P}) \subset \ker I(\mathfrak{P}')$.
page 181, line -1 (Christian Choffrut, March 2023).

$$P \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

page 182, line -5. the edge to wa...
page 185, line 20 (Christian Choffrut, March 2023). of the cylinders $[aa]$ and $[ba]$ is not ...
page 187, line 9. ...$+\psi(a_2a_3) + \ldots + \psi(a_kb_1)$.
page 187, line 11. ...$-\phi(a_2) + \ldots + \phi(b_1)$.
page 188, line 5. , $u \mapsto uv$,
page 194, ligne -1 (Felipe Arbulú, march 2022). $w = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$.
page 194, line -6 (Christian Choffrut, march 2023). in $\mathcal{R}_X(a)$
page 195, line 3 (Felipe Arbulú, march 2022).

$$N_2^k(\alpha v + \beta w) = 2^k\alpha v + (-1)^k\beta w = \begin{bmatrix} 2^k\alpha + (-1)^k\beta \\ 2^{k+1}\alpha + (-1)^{k+1}\beta \\ 3 \cdot 2^k\alpha \end{bmatrix}$$

page 195, line-10 (Christian Choffrut, march 2023). ...the unique state on ...
page 196, line -11 (Christian Choffrut, january 2023). (where M_a is the incidence matrix of the Rauzy automorphism L_a).
page 198, line 12 (Felipe Arbulú, march 2022). with positive cone $\mathbb{Z}_+[1/3]\times\mathbb{Z}$ and unit $(1,1)$.
page 204, line 3. if and only if $\mathcal{B}^1, \mathcal{B}^2$ have
page 209, line 15. ordered.

5 Chapter 5

page 214, line 3. $C_1 \supset C_2 \supset \cdots \supset C_n \supset \cdots$
page 214, line 9 (Simon Binder, jan 2023). $j_{n-1} + \ldots + j_1$
page 216, line 17 (Simon Binder, jan. 2023). $\sum_{1\le n\le n_0} j_n = j_{n_0} + \sum_{1\le n\le n_0-1} (h_{t_n}(n) - h_{t_{n-1}(n-1)}) =$...
page 219, line -1. Note that the dimension group of a stationary properly ordered BV system (X_E, V_E) is the direct limit of a stationary system defined by a primitive matrix. By Theorem 2.5.1, it has a unique state and thus (X_E, V_E) is uniquely ergodic.
page 222 line 9. Add: It is not obvious at all that the strong orbit equivalence is actually an equivalence relation between dynamical systems. This will result from Theorem 6.5.1.
page 238, add Exercise: A *pointed conjugacy* from (X, T, x) to (X', T', x') is a conjugacy ϕ from (X, T) to (X', T') such that $\phi(x) = x'$. Let $(V, E, \le)$ and $(V', E', \le')$ be two properly ordered Bratelli diagrams. Show that $(V, E, \le)$ and $(X', E', \le')$ have a common intertwinning if and only if there is a pointed conjugacy from $(X_E, T_E, x_{\min})$ to $(X_{E'}, T_{E'}, x'_{\min})$.

Solution: If $(V', E', \le')$ is obtained from $(V, E, \le)$ by telescoping, then the corresponding map $\phi\colon X_E \to X_{E'}$ is clearly a conjugacy such that $\phi(x_{min}) = x'_{\min}$.

6 Chapter 6

page 245, line 12, are the eventually constant sequences..
page 247, Figure 6.3 (Simon Binder, jan. 2923). ..of $(\mathbb{Z}_{(p_n)}, T)$.
page 252, line -5. Note that this implies that every minimal substitution shift is uniquely ergodic (see Section 5.3.4).
page 253, change Proposition 6.2.2 into *Let $\mathfrak{B} = (V, E, \leq)$ is a stationary Bratteli diagram. The morphism read on $(V, E, \leq)$ is primitive and eventually proper if and only if the diagram is properly ordered.*

Add to the proof, at the beginning: Assume first that $\mathfrak{B}$ is properly oredered. After the end : Conversely, assume that the morphism $\sigma\colon A^* \to A^*$ read on $\mathfrak{B}$ is primitive and eventually proper. Since σ is primitive, there is $n \geq 1$ such that $|\sigma^n(a)|_b > 0$ for every $a, b \in A$. Thus the Bratteli diagram (V, E) is simple. Next, since σ^n is left proper for some $n \geq 1$, there is a unique minimal path using all vertices $(nk, i^n(a))$ for $k \geq 0$. Similarly, there is a unique maximal path. Thus $(V, E, \leq)$ is properly ordered.

Add after the proof: Note that when $\mathfrak{B} = (V, E, \leq)$ is a stationary properly ordered Bratteli diagram, the point $x_{\min}$ of the BV-system (X_E, T_E) is the unique fixed point of the morphism read on $\mathfrak{B}$.
page 260, line -4 (Marie-Pierre Béal, december 2022). Indeed, $r(\phi \circ \tau^n(b))\ell = r(\sigma^n \circ \phi(b))\ell$ begins with $\sigma^n(\ell)$ (because $r\phi(b)\ell$ begins with $r\ell$).
page 293, line 11 (Marie-Pierre Béal, march 2022). the word $\varphi_u(j)u$ appears ....
page 293, line -2 (Marie-Pierre Béal, march 2022). we have $\varphi_u \circ \sigma_u^u =$...
page 295, line 2. Transfer 'Since y is not periodic' to the beginning of the next sentence.
page 296, line 9.

$$S^{|v|}(x) = ...$$

idem line 12 (twice).
page 302, Exercise 6.32. Show that for every sequence $x = x_0x_1 \cdots \in A^{\mathbb{N}}$, there are...such that $x = \lim \phi \circ \sigma_{x_0} \circ \cdots \circ \sigma_{x_n}(\#)$ and thus...

7 Chapter 8

page 367, line 5. $f(t) = 0$ otherwise.

8 Chapter 9

page 412, line 3. $k \geq 0$ and ...
page 413, line 6 (Christian Choffrut, march 2023).

$$n_i = \sum_{j=1}^{t} a_{ij} m_j$$

page 414, line -7. Consequently every isomorphism of C^*-algebras is an isomometry.

page 416, line -9 (Christian Choffrut, march 2023). are upper triangular with ...
page 420, line 4 (Christian Choffrut, march 2023). $q_{k+1} = a_{k+1}q_k + q_{k-1}$.
page 425, line -3 (Christian Choffrut, march 2023). since $\mathfrak{K}$ is
page 426, line -5. Add: Let $\alpha = [a_0; a_1, a_2, \ldots]$ be the continued fraction expansion of α and let p_n, q_n be the corresponding sequence of partial quotients (see the definition of a Sturmian algebra).
page 430, line -9. Let $\mathfrak{A} = \varinjlim \mathfrak{A}_m$ and $\mathfrak{B} = \varinjlim \mathfrak{B}_m$
page 427, line -13. $+(a_{n-1} + a_n)x^n + x^{n+1}$

9 Appendix A

page 435. line -6. (ii)$\Rightarrow$(i) Since T is onto, if the positive orbit of x is dense, the positive orbit of Tx is also dense. Indeed, since the positive orbit of x contains points arbitrary close to y, the positive orbit of Tx contains arbitrary close to x. Thus, if U, V are nonempty open sets, let $n \geq 0$ be such that $T^n x \in V$. Since $T^n x$ has dense positive orbit, there an m such that $T^{n+m}x \in V$ and thus $U \cap T^m V \neq \emptyset$.
page 462, line 5, $\phi \circ T^i(x) = \phi(x)$...
page 471, line -6. with positive cone $\mathbb{Z}_+[1/3] \times \mathbb{Z}$ and unit $(3, -1)$, which can be normalized to $(1, 1)$ through the automorphism $(\alpha, \beta) \mapsto (\alpha/3, -\beta)$.
page 484, line -3 (Christian Choffrut, february 2023) with right eigenvectors $u = [1\ 1]^t$ and $v = [1\ \ -2]^t$.
page 490, solution 6.32. Define σ_a by

$$\sigma_a(b) = \begin{cases} \#a & \text{if } b = \sharp \\ b & \text{otherwise} \end{cases}$$

and set $\phi(a) = a$ for every $a \in A$ and $\phi(\#) = \varepsilon$.

10 Appendix B

page 509, lines 12, 13 (Christian Choffrut, march 2023) F should be A (twice).
page 520, line 6. ...and, if M is irreducible, λ_M is the only eigenvalue with a nonnegative eigenvector.

11 Appendix C

page 536, line 3. with positive cone $\mathbb{Z}_+[1/3] \times \mathbb{Z}$

References

[1] Marie-Pierre Béal, Filippo Mignosi, Antonio Restivo, and Marinella Sciortino. Forbidden words in symbolic dynamics. *Advances in Applied Mathematics*, 25(2):163–193, 2000.

[2] Peter Walters. *An Introduction to Ergodic Theory*, volume 79 of *Graduate Texts in Mathematics*. Springer-Verlag, New York-Berlin, 1982.