Dimension groups and dynamical systems
Errata and complements

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1 Introduction

page 3, (Mike Boyle, February 2024), line 3. Delete Effros et al.
page 3, (Mike Boyle, February 2024), line 13-14. Replace ‘Cantor set’ by ‘minimal Cantor system’.
page 4, (Mike Boyle, February 2024), line -15. Replace ‘GP’ by ‘group’.

2 Chapter 1

page 7, (Eduardo Scarparo, April 2022). replace Example 1.1.1 by: As a simple example, consider $X = [0, 1]$, which is metric and compact as a closed interval of the real line $\mathbb{R}$. The transformation $T: x \mapsto x^2$ is a continuous map from $X$ onto $X$.
page 8, replace Example 1.1.2 by: Given $\alpha \in \mathbb{R}$, the transformation $T: x \mapsto x + \alpha \mod 1$ is not continuous at $x = 1 - \alpha \mod 1$ and thus the pair $([0, 1], T)$ is not a topological dynamical system. If we consider, instead of $[0, 1]$, the torus $T = \mathbb{R}/\mathbb{Z}$ in which 0 and 1 are identified, the transformation $T$ is simply the translation $T_\alpha: x \mapsto x + \alpha$ and becomes a homeomorphism on $T$. The system $(T, T_\alpha)$ is called the rotation of angle $\alpha$.
page 8, (Mike Boyle, February 2024), line 18. replace ‘a totally..’ by ‘a nonempty totally..’.
page 8, (Mike Boyle, February 2024), line -2. Replace ‘an isometry.’ by ‘an isometry if Card($A$) $\geq 2$.’
page 9, Proposition 1.1.3. (ii) $T$ is surjective and there is a point...
page 10, line -14 (Mike Boyle, February 2024), A topological dynamical system is minimal if it is nonempty and...
page 12, line 15. $f: X \to \mathbb{N}_+$.
page 12, line -4 (Mike Boyle, February 2024). Replace $X = [0, 1]$ by $X = \mathbb{R}/\mathbb{Z}$.
page 14, line -9. Add Note that in a one-sided shift space, the shift transformation is assumed to be surjective.
page 15, line -8 (Simon Binder, January 2023). $F_{n+2} = F_{n+1} + F_n$
page 16, line -8. Two measure-theoretic systems $(X, T, \mu)$ and $(X', T', \mu')$...
such that \( \varphi \circ T(x) = \varphi \circ T'(x) \) for every \( x \in X_1 \) and \( \mu(U) = \mu'(\varphi(U)) \) for every Borel subset \( U \subset X_1 \).

An integer \( p \) is a period of a sequence \( x \in A^\mathbb{N} \) if

\[ \begin{align*}
\varphi^n x \in [u]_X.
\end{align*} \]

Every return word to \( w \) is then a factor of the image by \( \sigma^n \) of a word of length at most equal to the maximal length of the return word to words of length 2.

The maximal length \( R \) of return words to words of length 2 is 8.

\((X^\varphi, T)\) is isomorphic to ...

Any dense subgroup \( G \) of \( \mathbb{R}^2 \) with the strict order, that is, with \( G^+ = \{(x, y) \in G \mid x > 0, y > 0\} \cup \{(0, 0)\} \) is a Riesz group. Indeed, let \( x_1, x_2, y_1, y_2 \in \mathbb{R}^2 \) with \( x_1 < y_1, y_2 \) and \( x_2 < y_1, y_2 \) as in ...

Any countable dense subgroup of \( \mathbb{R}^2 \) with the strict ordering is ...
We proceed by induction on the minimal number of generators of \( \ker(\alpha) \).

\( \eta \) is surjective morphism...

\( \eta' \) is surjective morphism...

\[ \alpha: (\mathbb{Z}_n, \mathbb{Z}_n^+) \rightarrow (G, G^+) \]

Change \( n \) to \( p \) (5 times).

All components of the corresponding eigenvector an eigenvector corresponding to \( \lambda \) can be chosen in the algebraic field ...

Replace the end of the proof by The maps \( p \) and \( q \) extend uniquely to \( \mathbb{Q} \)-linear forms (also called \( p, q \)) on \( \mathcal{R}_M \). These are restrictions of unique \( \mathbb{R} \)-linear forms \( P, Q \) on the smallest real vector space (\( W \), say) containing \( \mathcal{R}_M \). Because \( G_M \) is simple, by Prop. 2.2.9 we have \( p(x) > 0 \) if and only if \( q(x) > 0 \). By density of \( \mathbb{Q} \) in \( \mathbb{R} \), we have for all \( x \) in \( W \) that \( P(x) > 0 \) iff \( Q(x) > 0 \). It follows that \( P \) and \( Q \) are \( \mathbb{R} \)-vector space homomorphisms with the same kernel. Because \( P \) and \( Q \) also agree on the order unit, they are equal.

The statement may be false since there are primitive integer matrices with the same dominant eigenvalue \( \lambda \) for which the ideal classes coming for the eigenvectors for \( \lambda \) are different. It is shown in [3, Corollary 6.3] that there is only a finite number of nonisomorphic ordered groups \( G_M \) (with the unit not specified) when \( M \) is unimodular and primitive matrix with given dominant eigenvalue \( \lambda \). However, even when the ordered groups are isomorphic, the unital ordered groups could be distinct.

The statement may fail to be true if the system...

Let \( x_0 \) be a point with a dense positive orbit...

Since \( x_0 \) has a dense positive orbit...

The statement may fail to be true if the system...

Let \( (X, T) \) be a minimal Cantor system...

such that \( T^n x \) is in \( U_i \).
\( K^0(X, S) = \langle Z, Z_+, 2 \rangle \).

page 120, line 7, replace \( uA^a = uxA^a \) by \( [u] = [ux] \).

page 120, line -12 (Mike Boyle, February 2024). The image of \( C(X, \mathbb{R}_+) \) in ...

page 120. Corollary 3.3.5 (Mike Boyle, February 2024). Add Let \((X, T)\) be a minimal system.

page 128. Corollary 3.6.3 (Mike Boyle, February 2024). Add ‘Let \( X \) be a minimal shift space and \( \theta : X \to Y \) be the natural morphism onto the one-sided shift space associated with \( X \).’ Note that the statement is also true for a recurrent shift space because continuous functions are cohomologous to functions defined on positive coordinates.

page 132, line -8. such that \( \mu(T^n[u]) = \pi(u) \) for every \( n \in \mathbb{Z} \).

page 135, line -3 (Mike Boyle, February 2024). A direct proof of the fact that two Sturmian shifts of slopes \( \alpha, \beta \) are conjugate if and only if \( \alpha = \beta \) or \( \alpha = 1 - \beta \) can be found in [1] (see also [7, Theorem 5.19] where a proof using eigenvalues is given).

4 Chapter 4

page 159, line 6. Indeed,

page 164, line 4 (Simon Binder, January 2022). \( \ker I(\mathfrak{P}) \subset \ker I(\mathfrak{P}') \).

page 168, proof of Lemma 4.3.3 (Mike Boyle, February 2024). Reorder the proof by proving first (lines -13 to -6) that if the restriction of \( h \in C(X, Z) \) to \( Y \) is constant, then \( i(n)(f) = 0 \) for \( n \) large enough. Then define \( r \) as in lines 11 to 19 (the new first step guarantees that \( f \) does not depend on the choice of \( h \)).
Finally prove that $\alpha = 0$ if and only if $u$ is constant. The only if part has now been proved before.

page 181, line 4 (Mike Boyle, February 2024). The composition of a generalized path $p = (e_1^{\varepsilon_1}, \ldots, e_n^{\varepsilon_n})$ with $e_i \in E$ and $\varepsilon_i = \pm 1$ is $\kappa(p) = \varepsilon_1 e_1 + \ldots + \varepsilon_n e_n$.

page 181, line 8 (Mike Boyle, February 2024). Given by Equation (B.5).

page 181, line 9. is a basis of $C(\gamma)$. To show that $\rho$ is surjective, it is enough...

Since $\psi$ belongs to $Z_+^E$, this shows also that $\rho(Z_+^E) = C_+(\gamma)$, the isomorphism sends $C_+(\Gamma)$ onto $H_+(\Gamma)$.

page 181, line -1 (Christian Choffrut, March 2023).

$P \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

page 182, line -5. the edge to $wa$...

page 182, line -1 (Mike Boyle, February 2024). the set of cycles into the set of cycles.

page 185, line 20 (Christian Choffrut, March 2023). of the cylinders $[aa]$ and $[ba]$ is not ...

page 187, line 9. $+\psi(a_2 a_3) + \ldots + \psi(a_k b_1)$.

page 187, line 11. $-\phi(a_2) + \ldots + \phi(b_1)$.

page 188, line 5. $u \mapsto uv$.

page 194, ligne -1 (Felipe Arbulú, March 2022). $w = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$.

page 194, line -6 (Christian Choffrut, March 2023). in $R_X(a)$

page 195, line 3 (Felipe Arbulú, March 2022).

$N_2^k(\alpha v + \beta w) = \begin{bmatrix} 2^k \alpha + (-1)^k \beta \\ 2^{k+1} \alpha + (-1)^{k+1} \beta \\ 3 \cdot 2^k \alpha \end{bmatrix}$

page 195, line-10 (Christian Choffrut, March 2023). ...the unique state on ...

page 196, line -11 (Christian Choffrut, January 2023). (where $M_a$ is the incidence matrix of the Rauzy automorphism $L_a$).

page 198, line 12 (Felipe Arbulú, March 2022). with positive cone $Z_+[1/3] \times Z$ and unit $(1,1)$.

5 Chapter 5

page 204, line 3. if and only if $B^1, B^2$ have

page 209, line 15. ordered.

page 209, line 16. (Mike Boyle, February 2024). ... as a morphism (except for the ‘hat’ connecting the root to vertices at level 1).

page 210, line 16. (Mike Boyle, February 2024). for every $v \in V$. 5
C_1 \supset C_2 \supset \cdots \supset C_n \supset \cdots

\sum_{1 \leq n \leq n_0} j_n = j_{n_0} + \sum_{1 \leq n \leq n_0} (h_{n-1}(n) - h_{n-1}(n-1)) = \ldots

Note that the dimension group of a stationary properly ordered BV system \((X_E, V_E)\) is the direct limit of a stationary system defined by a primitive matrix. By Theorem 2.5.1, it has a unique state and thus \((X_E, V_E)\) is uniquely ergodic.

Two minimal invertible dynamical systems...

Indeed, in the basis the vectors,

\[ u = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad v = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \]

form a basis of \(\mathbb{Q}^2\) with \(\mathcal{R}_M = \mathbb{Q}u\) and \(\mathcal{K}_M = \mathbb{Q}v\). Since

\[ \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{3} u + \frac{2}{3} v \quad \text{and} \quad \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{3} u - \frac{1}{3} v \]

the projection \(P\) of \(\mathbb{R}^2\) onto \(\mathbb{R}u\) along \(\mathbb{R}v\) maps \(\mathbb{Z}^2\) onto \(\frac{1}{3} \mathbb{Z}u\). Because \(PM = MP\) and \(Pv = 0\), it follows that \(P\) defines an isomorphism mapping \((G_M, G^*_M, 1_M)\) onto \((\mathbb{Z}[1/3]u, \mathbb{Z}+[1/3]u, \frac{2}{3} u)\).

Add: It is not obvious at all that the strong orbit equivalence is actually an equivalence relation between dynamical systems. This will result from Theorem 6.5.1.

Example 5.5.4. (Mike Boyle, February 2024). Replace from the line in display. It has eigenvalues 1, 3 and eigenvectors

\[ u = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad v = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \]

corresponding to 3 and 1 respectively. The subgroup \(K = \mathbb{Z}u + \mathbb{Z}v\) has index two in \(\mathbb{Z}^2\), and \(\begin{bmatrix} 1 \\ 0 \end{bmatrix} \notin K\). Let \(B\) be the matrix defining the projection of \(\mathbb{Q}^2\) onto \(\mathbb{Q}u\) along \(\mathbb{Q}v\). Then \(B\) maps \(\mathbb{Z}^2\) onto \(\frac{1}{3} \mathbb{Z}u\). Also, \(BM = MB\), and \(B\) takes the order unit \(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\) to \(u\). It follows that \(B\) defines an isomorphism of unital ordered groups from \(G/\text{Inf}(G)\) to \(((1/2)\mathbb{Z}[1/3]u, (1/2)\mathbb{Z}+[1/3]u, u)\), which is isomorphic to \((\mathbb{Z}[1/3], \mathbb{Z}+[1/3], 2)\). One can check that \(G\), as an unordered group, is isomorphic to the group \(H = \mathbb{Z} \times \mathbb{Z}[1/3]\). Now, consider \(H\) as an ordered group with the strict order, given by \(H^+ = \{(x, y) \in H \mid y > 0\} \cup \{(0, 0)\}\). The ordered groups \((G, G^+), (H, H^+)\) are not isomorphic. Under an isomorphism, the infinitesimal subgroup \(\mathbb{Z}v\) of \(G\) would map to the infinitesimal subgroup of \(H\), which is \(\{(x, y) \in H \mid y = 0\}\), and the 3-divisible subgroup \(\mathbb{Z}[1/3]u\) of \(G\) would map to the 3-divisible subgroup of \(H\), which is \(\{(x, y) \in H \mid x = 0\}\). Thus an isomorphism would map \(\mathbb{Z}u + \mathbb{Z}v\) onto \(H\). This is a contradiction, because \(G\) contains elements outside \(\mathbb{Z}u + \mathbb{Z}v\).
By Theorem 5.5.3, the shift $X(\sigma)$... Also modify Figure 5.14 on the left as Figure 6.1 below (delete one edge at the first level).

Figure 5.1: Bratteli diagrams of orbit equivalent systems.

page 238, Exercise 5.12 (Mike Boyle, February 2024). stationary diagram with simple hat and with a $t \times t$...

page 238, add Exercise 5.14: A pointed conjugacy from $(X, T, x)$ to $(X', T', x')$ is a conjugacy $\phi$ from $(X, T)$ to $(X', T')$ such that $\phi(x) = x'$. Let $(V, E, \leq)$ and $(V', E', \leq')$ be two properly ordered Bratelli diagrams. Show that $(V, E, \leq)$ and $(X', E', \leq')$ have a common intertwining if and only if there is a pointed conjugacy from $(X_E, T_E, x_{min})$ to $(X'_E, T'_E, x'_{min})$.

Solution: If $(V', E', \leq')$ is obtained from $(V, E, \leq)$ by telescoping, then the corresponding map $\phi: X_E \to X_{E'}$ is clearly a conjugacy such that $\phi(x_{min}) = x_{min}'$.

page 239, Exercise 5.16. (Mike Boyle, February 2024). be minimal invertible topological ...

6 Chapter 6

page 245, line 12, are the eventually constant sequences..

page 247, Figure 6.3 (Simon Binder, Jan. 2923). ...of $(\mathbb{Z}_{(p_n)}, T)$.

page 247, line -15 (Mike Boyle, February 2024). ... a BV-representation with $pq^n$ edges at level $n$ with integers $p \geq 1$ and $q \geq 2$ such that there are $p$ edges at level 1 and $q$ edges at every other level, hence $pq^n$ paths from the root to vertices at level $n$ (see Exercise 6.8).

page 250, line -16 (Mike Boyle, February 2024). with $p = \text{Card}(r^{-1}\{u\})$.

page 252, line -5. Note that this implies that every minimal substitution shift is uniquely ergodic (see Section 5.3.4).

page 253, change Proposition 6.2.2 into Let $\mathcal{B} = (V, E, \leq)$ is a stationary Bratelli diagram. The morphism read on $(V, E, \leq)$ is primitive and eventually proper if and only if the diagram is properly ordered.
Add to the proof, at the beginning: Assume first that $\mathcal{B}$ is properly ordered.

After the end: Conversely, assume that the morphism $\sigma: A^* \to A^*$ read on $\mathcal{B}$ is primitive and eventually proper. Since $\sigma$ is primitive, there is $n \geq 1$ such that $|\sigma^n(a)|_b > 0$ for every $a, b \in A$. Thus the Bratteli diagram $(V, E)$ is simple. Next, since $\sigma^n$ is left proper for some $n \geq 1$, there is a unique minimal path using all vertices $(nk, i^n(a))$ for $k \geq 0$. Similarly, there is a unique maximal path. Thus $(V, E, \leq)$ is properly ordered.

Add after the proof: Note that when $\mathcal{B} = (V, E, \leq)$ is a stationary properly ordered Bratteli diagram, the point $x_{\text{min}}$ of the BV-system $(X_E, T_E)$ is the unique fixed point of the morphism read on $\mathcal{B}$.

page 255, line -1 (Mike Boyle, February 2024). is a $k \times \ell$-matrix $P$ with coefficients 0, 1 such that

(i) every column has exactly one coefficient equal to 1,

(ii) every row has at least one nonzero coefficient.

page 260, line -4 (Marie-Pierre Béal, December 2022). Indeed, $r(\phi \circ r^n(b))\ell = r(\sigma^n \circ \phi(b))\ell$ begins with $\sigma^n(\ell)$ (because $r\phi(b)\ell$ begins with $r\ell$).

page 273, Theorem 6.3.6 (Mike Boyle, February 2024). with topological rank $1 \leq k < \infty$

page 283, line -11 (Mike Boyle, February 2024). (as a factor of $x$) and period $q = |\tau_{[0,n]}(b)|$. Since $|u| = q + |\tau_{[0,n]}(b_0)| \geq p + q$, by Fine-Wilf...

page 292, line 6. (Mike Boyle, February 2024). the set of $u$-derivatives of $x$ for $u \in \mathcal{L}(x)$ is the same as the set of $u$-derivatives of $y$ for $u \in \mathcal{L}(y)$.

page 293, line 11 (Marie-Pierre Béal, March 2022). the word $\varphi_u(j)u$ appears ....

page 293, line -2 (Marie-Pierre Béal, March 2022). we have $\varphi_u \circ \sigma^n_u = ...$

page 295, line 2. Transfer ‘Since $y$ is not periodic’ to the beginning of the next sentence.

page 296, line 9.

S^{[u]}(x) = ...

idem line 12 (twice).

page 301 (Mike Boyle, February 2024). Exercise 6.25. is strong shift orbit equivalent...

page 301 (Mike Boyle, February 2024). Exercise 6.25. Hint: Show that the incidence matrix of $\sigma$ is $M(\varphi)^4$ where $\varphi$ is the Fibonacci morphism.

page 302 (Mike Boyle, February 2024). Exercise 6.30. ...a minimal shift space $X$ is linearly recurrent if and only if...

page 302, Exercise 6.32. Show that for every sequence $x = x_0x_1 \cdots \in A^\mathbb{N}$, there are...such that $x = \lim \phi \circ \sigma_{x_0} \circ \cdots \circ \sigma_{x_1}(\#)$ and thus...

page 304, (Mike Boyle, February 2024). Odometers are connected to solenoids (skew product of a Cantor set and a finite dimensional torus), see [6] or [2]. They are also called adding machines (Brown, 1976).

page 308 (Mike Boyle, February 2024). is proved to be a sufficient condition for unique ergodicity of minimal shift spaces in Boshernizan (1992).

page 310 (Mike Boyle, February 2024). Equation (7.1) should be $n - e = c$. 

8
Proposition 7.1.35. and distinct \( u, v \in V \).

Add: Since \( \sigma(A) \) is a tame basis, this shows also that not every tame basis can be contained in the language of a dendric shift. Thus the converse of Theorem 7.1.38 is false. A characterization of S-adic expansions of minimal dendric shifts is given in [5].

...is the largest root of \( x^3 - 4x^2 + 3x - 1 \). ...is also eventually dendric of characteristic 2.

7 Chapter 8

page 367, line 5. \( f(t) = 0 \) otherwise.

8 Chapter 9

page 412, line 3. \( k \geq 0 \) and ...

page 413, line 6 (Christian Choffrut, march 2023).

\[ n_i = \sum_{j=1}^{t} a_{ij}m_j \]

page 414, line -7. Consequently every isomorphism of \( C^*-\)algebras is an isometry.

page 416, line -9 (Christian Choffrut, march 2023). are upper triangular with ...

page 420, line 4 (Christian Choffrut, march 2023). \( q_{k+1} = a_{k+1}q_k + q_{k-1} \).

page 425, line -3 (Christian Choffrut, march 2023). since \( \mathcal{R} \) is page 426, line -5. Add: Let \( \alpha = [a_0; a_1, a_2, \ldots] \) be the continued fraction expansion of \( \alpha \) and let \( p_n, q_n \) be the corresponding sequence of partial quotients (see the definition of a Sturmian algebra).

page 430, line -6. (ii) \( \Rightarrow \) (i) To show that \( T \) is surjective, consider \( x \in X \) not in \( T(X) \). Since \( T(X) \) is closed, there is a neighborhood \( V \) of \( x \) whose closure does not intersect \( T(X) \). Let \( y \in T(X) \) and let \( U \) be a nonempty open set containing \( y \) whose closure does not intersect \( V \). Then, for every \( n \geq 0 \), \( U \cap T^{-n}V = \emptyset \), a contradiction.

Since \( T \) is onto, if the positive orbit of \( x \) is dense, the positive orbit of \( Tx \) is also dense. Indeed, since the positive orbit of \( x \) contains points arbitrary close
to $y$, the positive orbit of $Tx$ contains arbitrary close to $x$. Thus, if $U, V$ are nonempty open sets, let $n \geq 0$ be such that $T^n x \in V$. Since $T^n x$ has dense positive orbit, there an $m$ such that $T^{n+m} x \in V$ and thus $U \cap T^m V \neq \emptyset$.

page 462, line 5, $\phi \circ T^i(x) = \phi(x)$...

page 471, line -6, with positive cone $\mathbb{Z}_+[1/3] \times \mathbb{Z}$ and unit $(3, -1)$, which can be normalized to $(1, 1)$ through the automorphism $(\alpha, \beta) \mapsto (\alpha/3, -\beta)$.

page 484, line -3 (Christian Choffrut, February 2023) with right eigenvectors $u = [1 \ 1]^t$ and $v = [1 \ -2]^t$.

page 488, Solution 6.25 (Mike Boyle, February 2024). Since $\sigma$ differs from $\varphi^4$ by the orders of letters in $\sigma(0)$, we have $M(\sigma) = M(\varphi)^4$.

page 490, solution 6.32. Define $\sigma_a$ by

$$\sigma_a(b) = \begin{cases} \#a & \text{if } b = \# \\ b & \text{otherwise} \end{cases}$$

and set $\phi(a) = a$ for every $a \in A$ and $\phi(\#) = \varepsilon$.

10 Appendix B

page 507, line -4 $x^n + a_{n-1}x^{n-1} + \ldots$

page 508 (Mike Boyle, February 2024). Theorem B.1.2. In case 1, If $d = 2, 3 \mod 4$, the ring of integers...

page 509 (Mike Boyle, February 2024). line 3-5. Replace by: Thus the discriminant of every $\mathbb{Z}$-module basis of the algebraic integers of $K$ is the same nonzero integer $D$, called the discriminant of $K$.

page 509, lines 12, 13 (Christian Choffrut, March 2023) $F$ should be $A$ (twice).

page 520, line 6. ...and, if $M$ is irreducible, $\lambda_M$ is the only eigenvalue with a nonnegative eigenvector.

11 Appendix C

page 536, line 3. with positive cone $\mathbb{Z}_+[1/3] \times \mathbb{Z}$

12 Appendix D

page 543, $S$-adic conjecture. The problem has been solved in [4] who proved the following result. For a word $u$, root($u$) denotes the shortest prefix $v$ of $u$ such that $u = v^k$ for some $k$. For a set of words $W$, root($W$) = \{root($w$) | $w \in W$\}. 

10
Theorem 12.1 A minimal shift $X$ has at most linear complexity, that is, $X$ satisfies
\[ \limsup_{n \to \infty} \frac{p_X(n)}{n} < +\infty, \]
if and only if there exist $d > 0$ and an $S$-adic sequence $\sigma = (\sigma_n)_{n \geq 0}$ with $\sigma_n : A_{n+1}^* \to A_n^*$ nonerasing generating $X$ such that, for every $n \geq 0$, the following holds.

(P1) $\text{Card}\{\text{root}(\sigma_{[0,n]}(A_n))\} \leq d$.

(P2) $|\sigma_{[0,n]}(a)| \leq d \cdot |\sigma_{[0,n]}(b)|$ for every $a, b \in A_n$.

(P3) $|\sigma_{n-1}(a)| \leq d$ for every $a \in A_n$.

It is also proved in [4] that conditions (P1) and (P2) are equivalent to the weaker condition that $X$ has nonsuperlinear complexity, that is,
\[ \liminf_{n \to \infty} \frac{p_X(n)}{n} < +\infty, \]
Conditions (P1),(P2) and (P3) are related as follows to the natural conditions (FM) of the finiteness of the set morphisms $\sigma_n$ and and the condition (BA) that the cardinality of the alphabets $A_n$ are bounded.

Actually, (FM) implies both (BA) and Conditions (P1) and (P3), that (BA) implies (P1), and that, under (P3), (FM) and (BA) are equivalent.

It is shown in [4] there there exists a minimal shift $X$ with at most linear complexity such that every sequence $\tau$ generating $X$ and satisfying (P1),(P2),(P3) forms an infinite set of morphisms. Thus conditions (P1) and (P3) cannot be replaced by (FM).

page 543, line -1. A characterization of the $S$-adic representations of dendric shifts is given in [5].

References


