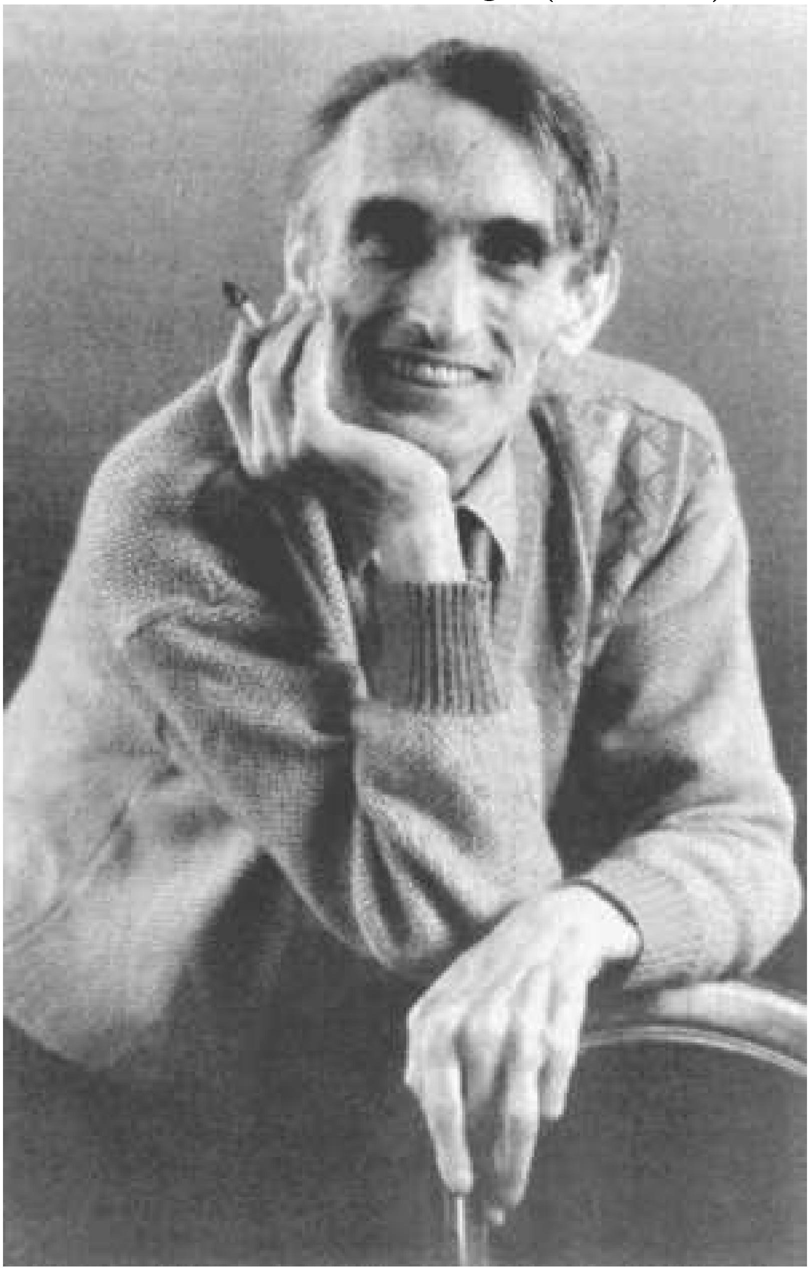
Marcel-Paul Schützenberger (1920-1996)



Marcel-Paul Schützenberger died on Monday, July 29, 1996. He was a great figure of science, leaving to mankind a bunch of fundamental discoveries and new ideas. He has worked during his life in an incredible number of areas and many of the people, including some of his former students, don't know in all its extensions this diversity and the depth of the results of all sorts that he had obtained. I am not sure to be well aware myself of all of them, such as his contribution to the discovery of the gene of trisomy in the early 50.

I will concentrate here on his work in Theoretical Computer Science. You may find in the electronic Journal of Combinatorics at

(http://ejc.math.gatech.edu:8080/Journal/Volume_3/Html/v3i1f1.html

contributions by Herbert Wilf and by Dominique Foata more focused on his work in combinatorics. Another source of additional information is the volume *Mots* (by Lothaire, Hermes, 1990). It is a collection of papers dedicated to Marco Schützenberger by his friends and former students. You will find there a bibliography (compiled by Imre Simon) of 155 scientific papers by Marco.

The starting point of Marco's work in Computer Science was the theory of variable length codes. He published in 1955 a paper ('Une théorie algébrique du codage') presented at the algebra seminar in Paris which already contains many of the ideas of his work on automata. For example, one finds in this paper the definition of the syntactic semigroup, of recognizable sets and their equality with rational sets, actually almost simultaneously with the appearance of Kleene's work. The problem of understanding the nature of the property of unique decipherability fascinated him since the beginning. He found there an incredible interplay between algebra through the use of finite semigroups, probability theory and combinatorics. Many of his favourite subjects were put together. Later he published a series of results on variable-length codes all of them reported in our book with Jean Berstel (*Theory of Codes*, Academic Press, 1984). To single out just one of them, I would quote the theorem expressing that a finite maximal code either is prefix or has an infinite deciphering delay (*J. Comb. Th.*, **1**, 1966, 437-442). His ideas on codes lead him later on to several deep results on rational functions and transducers.

It is also very early that Marco Schützenberger began to work on context-free grammars. The famous Chomsky-Schützenberger theorem asserting that any context-free language is a coding of a simple Dyck language appeared in 1963. His fundamental idea was that context-free languages were a non-commutative version of algebraic series, in the same way as finite-state languages are the non-commutative counterpart of rational series. His work on context-free grammars is thus closely related to the ones on algebraic and rational series. He would in this vein publish in 1962 a paper in which he proves a version for series of the property of the family of context-free languages to be closed under intersection with a rational languages, thus appearing as a non-commutative version of a theorem on the Hadamard product of an algabraic series with a rational one (On a theorem of R. Jungen, *Proc. Amer. Math. Soc.*, *13*, 885-890).

The beautiful theorem on aperiodic and star-free sets was published in 1965 (On finite monoids having only trivial subgroups, *Inf. and Control*, **8**, 190-194). It is the first result of the theory of varieties that he would develop with S. Eilenberg and appears in the second volume of *Automata, Languages and Machines* (Academic Press, 1976).

A good part of the rest of his mathematical work is either in algebra or in combinatorics.

In algebra, he has deeply influenced the theory of semigoups. The name of Schützenberger group of a $\mathcal{D}$-class was coined by A. H. Clifford in the first book on semigroup theory (A. H. Clifford and G. B. Preston, *The algebraic Theory of Semigroups*, Amer. Math. Soc., 1961). The techniques that he had developped in semigroups were in many cases his 'arme secrète' for his work on automata. For example, the theorem on star-free sets mentionned above can be easily stated without reference to semigroups but I don't know of a proof that does not use finite semigroups (a fact which might explain that this result is not as well-known as one could expect). Another part of Marco's work in algebra is on Lie algebras. This is again related with codes and automata. Actually, one of his most beautiful results relates Lie algebras decompositions with factorizations of the free monoid (On a factorization of free monoids, *Proc. Amer. Math. Soc.*, **16**, 1965, 21-24).

His contributions in combinatorics are probably as important as those in computer science, making Marco, to quote Herbert Wilf, 'one of the most creative and influential combinatorialists of this century'. It is probably his work on Young tableaux which is most famous. Some of his early work is reported in Knuth's volume 3 (*Sorting and Searching*, Addison Wesley, 1975). He was not so far from his grass roots since he introduced there, in his further work with Alain Lascoux, a semigroup (the *plactic monoid*) which reveals many properties of the tableaux. He was also the inventor of *bijective proofs*. The rough idea is to prove combinatorial identities by exhibiting a bijection between two sets enumerated by the two sides of the identity. The sets were often formal languages as for the case of the description of graphs by context-free languages.

Marco Schützenberger was without doubt the founder of combinatorics on words. One of the references for his pioneering work there is the collective book by Lothaire published in 1983 and in which he wrote himself a chapter, together with the group of his former students in this field (*Combinatorics on Words*, Cambridge University Press). He used to say that I was the "protonotaire" of Lothaire. Having accepted since years this mysterious title, I had today the curiosity to look in a dictionnary. Here is what the Robert says:

> **PROTONOTAIRE** n. m. – 1390; lat. ecclés. *protonotarius*, de proto-, et lat. *notarius*. → Notaire
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> ◊ **1.** Prélat de la cour romaine, du rang le plus élevé parmi ceux qui n'ont pas le caractère épiscopal. *Les protonotaires apostoliques participants* (de numero participiendum), *officiers de la cour pontificale, sont constitués en collège et sont chargés d'enregistrer les actes pontificaux dans les circonstances solennelles, à la congrégation des rites, de signer les bulles...–Protonotaires à l'instar* (ad instar participantium). *Protonotaires titulaires, honoraires* ou *protonotaires noirs* (dont l'habit ne comporte pas les ornements amarante des autres protonotaires).
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> ◊ **2.** (1680). Hist. Premier notaire d'un empereur romain.

(1869). Dignitaire laïc du moyen-âge (chef de la chancellerie, etc.)
◊ **3.** (Canada, 1795). Fonctionnaire chargé de l'enregistrement des actes dans un bureau régional.
DÉR. Protonotariat

I give up any attempt to translate the definition in english.

Much of his work in mathematics, as we have seen, is connected with automata, words and coding. It would certainely be difficult to tell whether Marco Schützenberger has applied mathematics to computer science or conversely. He once told me long ago that his work consisted in applying his intuitions from data processing to bring new contributions to mathematics. Was he really joking? He had actually especially strong beliefs and a passion for discussion if not for controversy. Among the favourite victims of his irony were all sorts of fools including the tenants of artificial intelligence, as they were trying to deny, would he say, the difference between men and women.

So much for mathematics. His contributions will live for a long time. A lot of the rest will not survive his numerous friends and students. All of them remember long passionating discussions on all possible subjects. The influence that he had on each of us goes well beyond our mathematical education. We all have lost a close and very affectionate friend who hided behind bitter irony and an immederate love for paradoxes, an incredibly generous nature.

Dominique Perrin