Reverse deterministic automata

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Outline

- Codeterministic automata
- Prophetic automata
- A surprising method

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CODETERMINISTIC AUTOMATA ON INFINITE WORDS

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We prove that any recognizable set of infinite words is the infinite behaviour of some finite codeterministic automaton.

Keywords: Infinite word, finite automaton

1. Introduction

There are several possibilities for the definition of acceptance of an infinite word by a finite automaton. One of them, often referred to as *Buchi's condition*, requires that, for some computation, the automaton goes infinitely often through a final state. Another one, known as *Muller's condition* requires that, for some computation, the set of states reached infinitely often belongs to some prescribed family of sets of states. These two conditions can be proved to be equivalent in the sense that, given a finite automaton accepting a set of infinite words with Muller's condition, one may construct another accepting the same set with Buchi's condition and vice versa. The important difference between both conditions lies in their computations on a given infinite word is finite (and bounded by the number of states).

The above result was previously announced by Mostowski [3] but we were not able to understand his proof. Our own motivations for proving this result are related with the study of two-sided infinite words initiated in [2]. The presentation given here is mainly self-contained. The notation and the proof of some elementary results needed can be found in [1].

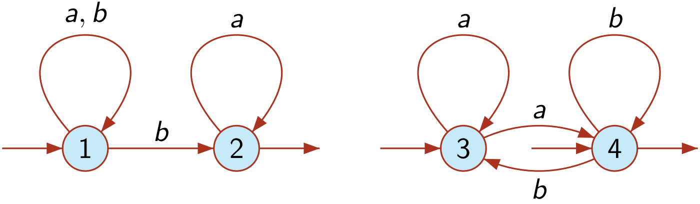
2. Preliminaries

Let A denote a (finite or infinite) alphabet. We denote by A^* the set of finite words over the alphabet A. The empty word is denoted by 1 and

Theorem (Mostowski (1982), D. Beauquier and D.P. (1985))

Any recognizable set of ω-words can be recognized by a codeterministic automaton.

Example: $a^* b\{a, b\}^\omega$.



Prophetic automata

Any word is the label of a unique path with an infinitely repeated final state.

Theorem (Carton, Michel, 2000)

Any recognizable set can be recognized by a prophetic automaton.

Link with temporal logic and timed automata.

A surprising method (B. Girod, 1999)

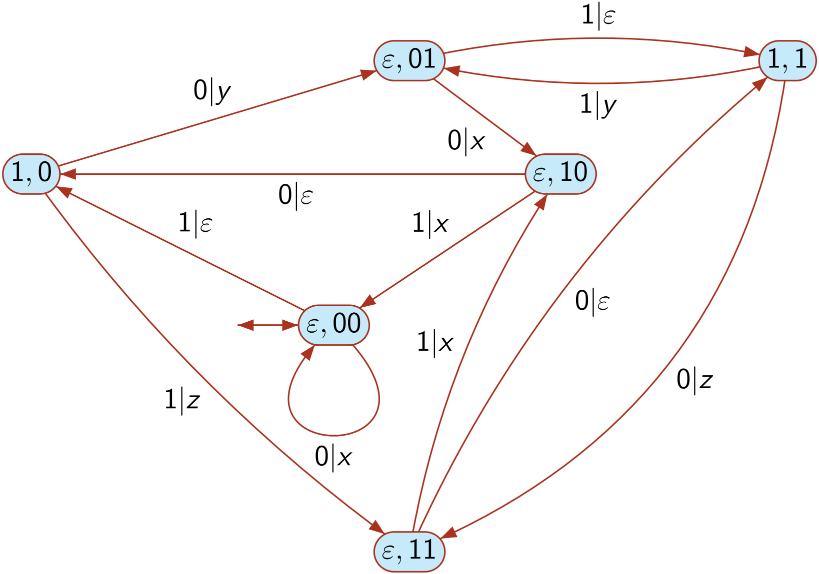
Consider a binary prefix code X. Encode a sequence of bits as follows:

Example

$x = 0, y = 10, z = 11$

x	y		z		x		
0	1	0	1	1	0	0	0
0	0	0	0	1	1	1	0
0	1	0	1	0	1	1	0

The decoder



$\varepsilon, 01$
$1, 1$
$1, 0$
$\varepsilon, 10$
$\varepsilon, 00$
$\varepsilon, 11$
$1|\varepsilon$
$1|y$
$0|y$
$0|x$
$0|\varepsilon$
$1|\varepsilon$
$1|x$
$0|\varepsilon$
$1|x$
$0|z$
$1|z$
$0|x$