Haskell Types and Typeclasses

http://igm.univ-mlv.fr/~vialette/?section=teaching

Stéphane Vialette

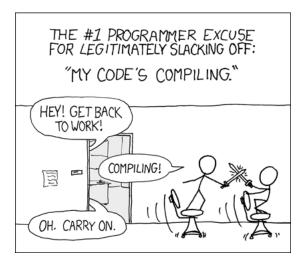
LIGM, Université Gustave Eiffel

November 21, 2022



< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □

Believe the type







Believe the type

One of Haskell's greatest strenghts is its powerful type system.

In Haskell, every expression's type is known at compile time, which leads to safer code.

Haskell has type inference.





Explicit type declaration

Explicit types are always denoted with the first letter in capital case

```
\begin{split} \lambda: :type \ True \\ True :: \ Bool \\ \lambda: :type \ 'a' \\ 'a' :: \ Char \\ \lambda: :type \ "hello" \\ "hello" :: \ [Char] \\ \lambda: :type \ (True, \ 'a', \ "hello") \\ (True, \ 'a', \ "hello") :: \ (Bool, \ Char, \ [Char]) \end{split}
```





Explicit type declaration

```
\lambda: :type 1 == 2

1 == 2 :: Bool

\lambda: :type 1

1 :: Num a => a

\lambda: :type 1.0

1.0 :: Fractional a => a

\lambda: :type (1/0)

(1/0) :: Fractional a => a
```





Explicit type declaration

Functions have also types.

-- filter out lowercase letters
remNonUpperCase :: [Char] -> [Char]
remNonUpperCase s = [c | c <- s, c `elem` ['A'...'Z']]</pre>

-- add three integers addThree :: Int -> Int -> Int -> Int addThree x y z = x + y + z





Int stands for integers.

Int is bounded which means that it has a minimum value and a maximum value.

Integer is also used to store integers, but it is not bounded.

```
factorial :: Int -> Int
factorial n = product [1..n]
```

 $\begin{array}{l} \lambda: \text{ factorial } 40 \\ -70609262346240000 \\ \lambda: \end{array}$





Int stands for integers.

Int is bounded which means that it has a minimum value and a maximum value.

Integer is also used to store integers, but it is not bounded.

```
factorial :: Integer -> Integer
factorial n = product [1..n]
```

```
\lambda: :type product
product :: (Foldable t, Num a) => t a -> a
\lambda: factorial 40
815915283247897734345611269596115894272000000000
```





Int stands for integers.

Int is bounded which means that it has a minimum value and a maximum value.

Integer is also used to store integers, but it is not bounded.

```
factorial n = product [1..n]
```

```
\lambda: :type factorial
factorial :: (Enum a, Num a) => a -> a
\lambda: factorial 40
815915283247897734345611269596115894272000000000
```





Float is a real floating point with single precision.

circumference :: Float -> Float
circumference r = 2 * pi * r

 λ : circumference 4.0 25.132742





Double is a real floating point with double the precision!

circumference :: Double -> Double
circumference r = 2 * pi * r





Type inference.

circumference :: Floating a => a -> a
circumference r = 2 * pi * r





Type variables

 λ : :type head head :: [a] -> a

Because a is not in capital case it's actually a type variable.

That means that a can be of any type.

This is much like generics in other languages, only in Haskell it's much more powerful because it allows us to easily write very general functions if they don't use any specific behavior of the types in them.

Functions that have type variables are called **polymorphic func-tions**.

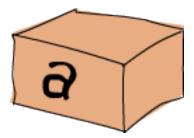




Type variables

 λ : :type head head :: [a] -> a

The type declaration of head states that it takes a list of any type and returns one element of that type.







Э

・ロト ・ 同ト ・ ヨト ・ ヨト

Type variables

$$\lambda$$
: :type fst
fst :: (a, b) -> a
 λ : :type snd
snd :: (a, b) -> b

Note that just because a and b are different type variables, they don't have to be different types.

It just states that the first component's type and the return value's type are the same.











A **typeclass** is a sort of *interface* that defines some behavior.

If a type is a part of a typeclass, that means that it supports and implements the behavior the typeclass describes.

A lot of people coming from OOP get confused by typeclasses because they think they are like classes in object oriented languages. They're not. You can think of them kind of as Java interfaces, only better.





What's the type signature of the == function?

λ: :type (==) (==) :: Eq a => a -> a -> Bool

Everything before the => symbol is called a **class constraint**.

We can read the previous type declaration like this:

The equality function takes any two values that are of the same type and returns a Bool. The type of those two values must be a member of the Eq class (this was the class constraint).





What's the type signature of the == function?

λ: :type (==) (==) :: Eq a => a -> a -> Bool

Everything before the => symbol is called a **class constraint**.

The Eq typeclass provides an interface for testing for equality.

Any type where it makes sense to test for equality between two values of that type should be a member of the Eq class.

All standard Haskell types except for IO (the type for dealing with input and output) and functions are a part of the Eq typeclass.



What's the type signature of the == function?

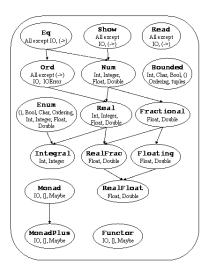
λ: :type (==) (==) :: Eq a => a -> a -> Bool

Everything before the => symbol is called a class constraint.

The elem function has a type of $(Eq a) \Rightarrow a \Rightarrow [a] \Rightarrow Bool because it uses == over a list to check whether some value we're looking for is in it.$











Basic typeclasses

Eq is used for types that support equality testing.

The functions its members implement are == and /=.

So if there's an Eq class constraint for a type variable in a function, it uses == or /= somewhere inside its definition.

All the types we mentioned previously except for functions are part of Eq, so they can be tested for equality.





Basic typeclasses

Eq is used for types that support equality testing.

```
\lambda: 7 == 7

True

\lambda: 7 /= 7

False

\lambda: 'a' == 'a'

True

\lambda: "Hello" == "Hello"

True

\lambda: 3.432 == 3.432

True
```





Basic typeclasses

Eq is used for types that support equality testing.

class Eq a where
 (==) :: a -> a -> Bool
 (/=) :: a -> a -> Bool
 x /= y = not (x == y)





Basic typeclasses

Ord is for types that have an ordering.

```
λ: :type (>)
(>) :: Ord a => a -> a -> Bool
```

All the types we covered so far except for functions are part of Ord.

Ord covers all the standard comparing functions such as >, <, >= and <=.

The compare function takes two Ord members of the same type and returns an ordering.

Ordering is a type that can be GT, LT or EQ, meaning greater than, lesser than and equal, respectively.

Basic typeclasses

Ord is for types that have an ordering.

```
\lambda: "Abrakadabra" < "Zebra" 
True 
\lambda: 5 >= 2 
True
```





Basic typeclasses

Ord is for types that have an ordering.





Basic typeclasses

Ord is for types that have an ordering.

```
class (Eq a) => Ord a where
 (<) :: a -> a -> Bool
 (<=) :: a -> a -> Bool
 (>=) :: a -> a -> Bool
 (>) :: a -> a -> Bool
 max :: a -> a -> Bool
 min :: a -> a -> a
```





Basic typeclasses

Members of Show can be presented as strings.

All types covered so far except for functions are a part of Show.

The most used function that deals with the Show typeclass is show. It takes a value whose type is a member of Show and presents it to us as a string.





Basic typeclasses

Members of Show can be presented as strings.

```
λ: :type show
show :: Show a => a -> String
λ: show 5
"5"
λ: show 'a'
"'a'"
λ: show "toto"
"\"toto\""
λ: show True
"True"
```





Basic typeclasses

Members of Show can be presented as strings.

class Show a where
 show :: a -> String

-- showList and showsPrec omitted





Basic typeclasses

Read is sort of the opposite typeclass of Show.

The **read** function takes a string and returns a type which is a member of **Read**.

```
\lambda: read "True" || True
True
\lambda: read "1" + 2
3
\lambda: read "1.2" * 3.4
4.08
\lambda: read "[1,2,3,4]" ++ [5]
[1,2,3,4,5]
```





Basic typeclasses

Read is sort of the opposite typeclass of Show.

The **read** function takes a string and returns a type which is a member of **Read**.





Basic typeclasses

Read is sort of the opposite typeclass of Show.

The **read** function takes a string and returns a type which is a member of **Read**.

```
\lambda: :type read
read :: Read a => String -> a
```

It returns a type that's part of **Read** but if we don't try to use it in some way later, it has no way of knowing which type. That's why we can use explicit type annotations.

Type annotations are a way of explicitly saying what the type of an expression should be. We do that by adding :: at the end of the expression and then specifying a type.





- 日本 - 4 国本 - 4 国本 - 日本

Basic typeclasses

Read is sort of the opposite typeclass of Show.

The **read** function takes a string and returns a type which is a member of **Read**.

```
\lambda: read "True"::Bool
True
\lambda: read "1"::Int
1
\lambda: read "1.2"::Float
1.2
\lambda: read "[1,2,3,4]"::[Int]
[1,2,3,4]
```





Basic typeclasses

Enum members are sequentially ordered types – they can be enumerated.

The main advantage of the Enum typeclass is that we can use its types in list ranges.

They also have defined successors and predecesors, which you can get with the succ and pred functions.

Types in this class: (), Bool, Char, Ordering, Int, Integer, Float and Double.





Basic typeclasses

Enum members are sequentially ordered types – they can be enumerated.

```
\lambda: ['a'..'e']
"abcde"
\lambda: [LT ... GT]
[LT,EQ,GT]
\lambda: [3 .. 5]
[3, 4, 5]
\lambda: succ 'a'
'b'
\lambda : succ LT
EQ
\lambda: succ 1
2
```



Basic typeclasses

Enum members are sequentially ordered types – they can be enumerated.

class Enum a where

succ, pred	:: a -> a	
toEnum	:: Int -> a	
fromEnum	:: a -> Int	
enumFrom	:: a -> [a]	[n]
enumFromThen	:: a -> a -> [a]	[n,n']
enumFromTo	:: a -> a -> [a]	[nm]
enumFromThenTo	:: a -> a -> a -> [a]	[n,n'm]

< ロ > < 目 > < 目 > < 目 > < 目 > < 目 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <



Basic typeclasses

Bounded members have an upper and a lower bound.

minBound and maxBound are interesting because they have a type of (Bounded a) => a. In a sense they are polymorphic constants.

All tuples are also part of **Bounded** if the components are also in it.





Basic typeclasses

Bounded members have an upper and a lower bound.

```
\lambda: maxBound :: Char

'\1114111'

\lambda: maxBound :: Bool

True

\lambda: minBound :: Bool

False

\lambda: maxBound :: (Bool, Int, Char)

(True,9223372036854775807,'\1114111')
```





Basic typeclasses

Bounded members have an upper and a lower bound.

class Bounded a where minBound :: a maxBound :: a





Basic typeclasses

Num is a numeric typeclass. Its members have the property of being able to act like numbers.

Whole numbers are also polymorphic constants. They can act like any type that's a member of the Num typeclass.

```
\lambda: :type 5
5 :: Num a => a
```





Basic typeclasses

Num is a numeric typeclass. Its members have the property of being able to act like numbers.

 λ : 20 :: Int 20 λ : 20 :: Integer 20 λ : 20 :: Float 20.0 λ : 20 :: Double 20.0





Basic typeclasses

Num is a numeric typeclass. Its members have the property of being able to act like numbers.

```
λ: :type (*)
(*) :: Num a => a -> a -> a
```

It takes two numbers of the same type and returns a number of that type. That's why (5 :: Int) * (6 :: Integer) will result in a type error whereas 5 * (6 :: Integer) will work just fine and produce an Integer because 5 can act like an Integer or an Int.

To join Num, a type must already be friends with Show and Eq.





Basic typeclasses

Integral is also a numeric typeclass.

Num includes all numbers, including real numbers and integral numbers, Integral includes only integral (whole) numbers. In this typeclass are Int and Integer.

```
class (Real a, Enum a) => Integral a where
quot, rem, div, mod :: a -> a -> a
quotRem, divMod :: a -> a -> (a,a)
toInteger :: a -> Integer
```





Basic typeclasses

A very useful function for dealing with numbers is **fromIntegral**.

It has a type declaration of

fromIntegral :: (Num b, Integral a) => a -> b.

From its type signature we see that it takes an integral number and turns it into a more general number.

That's useful when you want integral and floating point types to work together nicely.





Basic typeclasses

A very useful function for dealing with numbers is **fromIntegral**.

For instance, the length function has a type declaration of length :: [a] -> Int instead of having a more general type of (Num b) => length :: [a] -> b. I think that's there for historical reasons or something, although in my opinion, it's pretty stupid.

If we try to get a length of a list and then add it to 3.2, we'll get an error because we tried to add together an Int and a floating point number. So to get around this, we do fromIntegral (length [1,2,3,4]) + 3.2 and it all works out.

