

# Functional programming

## Lecture 01 — First steps

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# Functional programming concepts

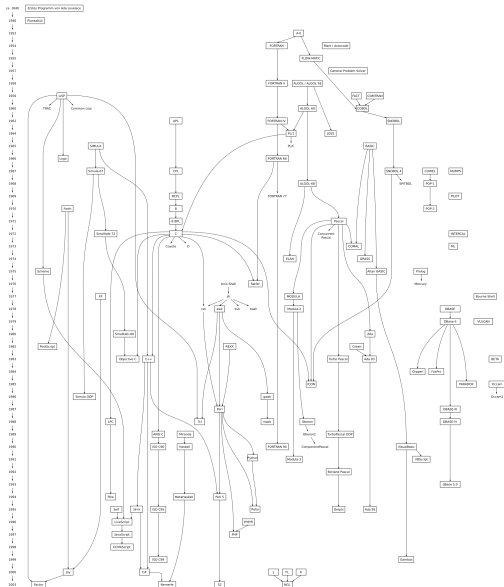
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Functional programming concepts

First steps

Types and classes

# Genealogy of programming languages



# Functional languages



# Main functional programming languages



## Lisp

Lisp (historically, LISP) is a family of computer programming languages with a long history and a distinctive, fully parenthesized prefix notation. Originally specified in 1958, Lisp is the second-oldest high-level programming language in widespread use today. (Only Fortran is older, by one year.)



## Erlang

Erlang is a general-purpose, concurrent, functional programming language, as well as a garbage-collected runtime system.



## Elixir

Elixir is a functional, concurrent, general-purpose programming language that runs on the Erlang virtual machine (BEAM).

# Main functional programming languages



**F#**

F# is a strongly typed, multi-paradigm programming language that encompasses functional, imperative, and object-oriented programming methods. It is being developed at Microsoft Developer Division and is being distributed as a fully supported language in the .NET framework.





## Ocaml

Ocaml, originally named Objective Caml, is the main implementation of the programming language Caml. OCaml's toolset includes an interactive top-level interpreter, a bytecode compiler, a reversible debugger, a package manager (OPAM), and an optimizing native code compiler.

# Main functional programming languages



## Clojure

Clojure is a dialect of the Lisp programming language. Clojure is a general-purpose programming language with an emphasis on functional programming. It runs on the Java virtual machine and the Common Language Runtime.

# Main functional programming languages



## Racket

Racket, formerly PLT Scheme, is a general purpose, multi-paradigm programming language in the Lisp-Scheme family. One of its design goals is to serve as a platform for language creation, design, and implementation

# Main functional programming languages



## Elm

Elm is a domain-specific programming language for declaratively creating web browser-based graphical user interfaces. Elm is purely functional, and is developed with emphasis on usability, performance, and robustness.



## Scala

Scala is a general-purpose programming language providing support for functional programming and a strong static type system. Designed to be concise, many of Scala's design decisions aimed to address criticisms of Java.

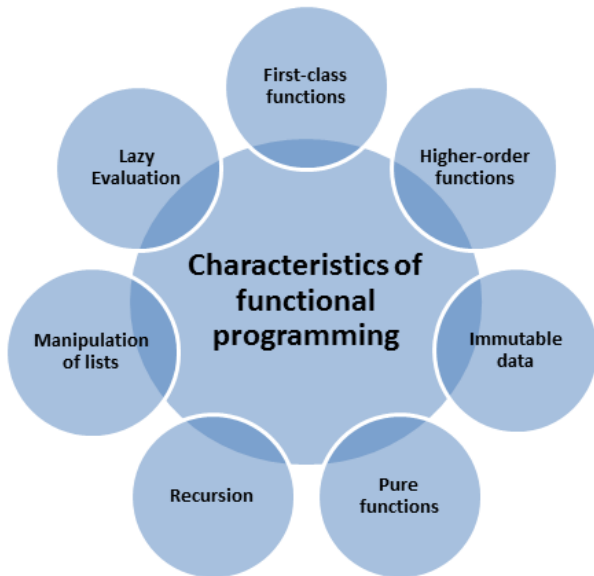
# Main functional programming languages



## Haskell

Haskell is a general-purpose, statically-typed, purely functional programming language with type inference and lazy evaluation. Designed for teaching, research and industrial applications, Haskell has pioneered a number of programming language features such as type classes, which enable type-safe operator overloading, and monadic IO. Haskell's main implementation is the Glasgow Haskell Compiler (GHC). It is named after logician Haskell Curry.

# Characteristics of functional programming



# Haskell

- Haskell is a compiled, statically typed, functional programming language.
- It was created in the early 1990s as one of the first open-source purely functional programming languages.
- It is named after the American logician Haskell Brooks Curry.





- Concise programs
- Powerful type system
- List comprehensions
- Recursive functions
- High-order functions
- Effectful functions
- Generic functions
- Lazy evaluation
- Equational reasoning

## The imperatives

- **GHC**: state-of-the-art, open source, compiler and interactive environment for the functional language Haskell.
- **GHCi**: GHC's interactive environment.
- **Hackage**: Haskell community's central package archive of open source software.

## Testing Frameworks

- **QuickCheck**: powerful testing framework where test cases are generated according to specific properties.
- **HUnit**: unit testing framework similar to JUnit.
- **Hspec**: a testing framework similar to RSpec with support for QuickCheck and HUnit.
- **The Haskell Test Framework, HTF**: integrates both Hunit and QuickCheck.

## Ancillary Tools

- **darcs**: revision control system.
- **haddock**: documentation system.
- **cabal**: build system.
- **stack**: build system.
- **hoogle**: type-aware API search engine.

## Static Analysis Tools

- **hlint**: detect common style mistakes and redundant parts of syntax, improving code quality.
- **Sourcegraph**: Haskell visualizer.

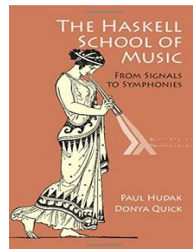
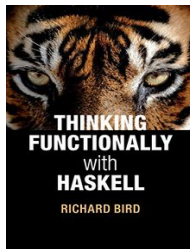
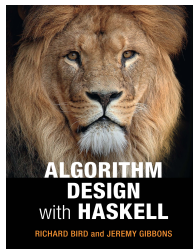
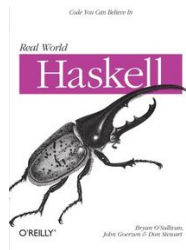
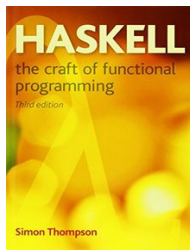
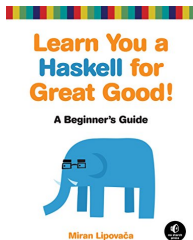
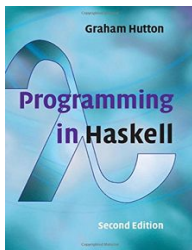
## Dynamic Analysis Tools

- **criterion**: powerful benchmarking framework.
- **hpc**: check evaluation coverage of a haskell program, useful for determining test coverage.

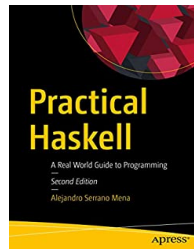
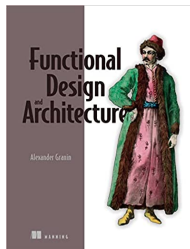
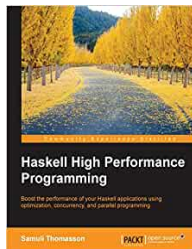
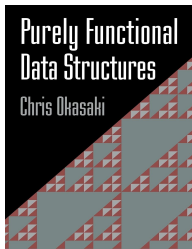
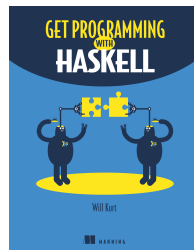
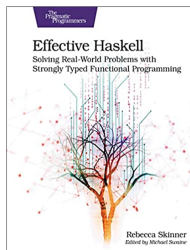
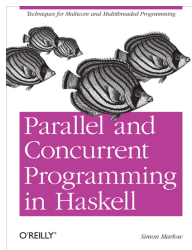
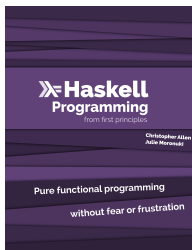
## IDEs

- VSCodium.
- IntelliJ.
- Vim.
- GNU Emacs.
- Haskell for Mac (commercial).
- Sublime Text (commercial)

# Haskell books

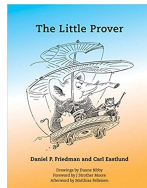
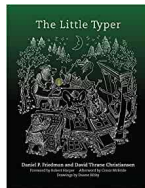
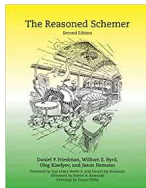
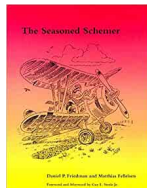
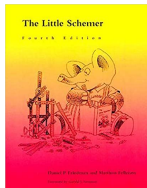
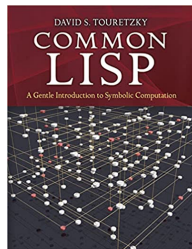
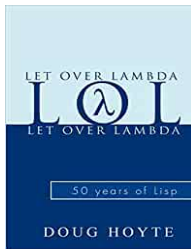
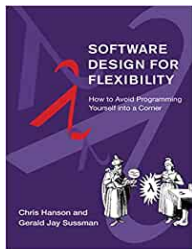
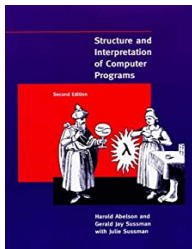


# Haskell books





# Functional programming books



## A taste of haskell

```
sum :: Num a => [a] -> a
sum []      = 0
sum (x : xs) = x + sum xs
```

# A taste of haskell

```
sum :: Num a => [a] -> a
sum []          = 0
sum (x : xs)    = x + sum xs
```

```
    sum [1,2,3]
=   { applying function sum }
    1 + sum [2,3]
=   { applying function sum }
    1 + 2 + sum [3]
=   { applying function sum }
    1 + 2 + 3 + sum []
=   { applying function sum }
    1 + 2 + 3 + 0
=   { applying function +   }
    3 + 3 + 0
=   { applying function +   }
    6 + 0
=   { applying function +   }
    6
```

## A taste of haskell

```
qsort :: Ord a => [a] -> [a]
qsort [] = []
qsort (x : xs) = qsort smaller ++ [x] ++ qsort larger
  where
    smaller = [x' | x' <- xs, x' <= x]
    larger  = [x' | x' <- xs, x' > x]
```

## A taste of haskell

```
qsort :: Ord a => [a] -> [a]
qsort [] = []
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    where
        smaller = [x' | x' <- xs, x' <= x]
        larger  = [x' | x' <- xs, x' > x]

qsort [x]
= { applying function qsort }
  qsort [] ++ [x] ++ qsort [x]
= { applying function qsort }
  [] ++ [x] ++ []
= { applying function ++ (twice) }
  [x]
```

# A taste of haskell

```
qsort :: Ord a => [a] -> [a]
qsort [] = []
qsort (x : xs) = qsort smaller ++ [x] ++ qsort larger
  where
    smaller = [x' | x' <- xs, x' <= x]
    larger  = [x' | x' <- xs, x' > x]

qsort [3,5,1,4,2]
= { applying function qsort }
  qsort [1,2] ++ [3] ++ qsort [5,4]
= { applying function qsort (twice) }
  (qsort [] ++ [1] ++ qsort [2]) ++ [3] ++ (qsort [4] ++ [5] ++ qsort [])
= { applying function qsort (four times) }
  ([ ] ++ [1] ++ [2]) ++ [3] ++ ([4] ++ [5] ++ [ ])
= { applying function ++ (four times) }
  [1,2] ++ [3] ++ [4,5]
= { applying function ++ (twice) }
  [1,2,3,4,5]
```

# First steps

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Functional programming concepts

First steps

Types and classes

- The *Glasgow Haskell Compiler* (GHC) is the state-of-the-art open source implementation of Haskell
- The GHC is freely available for a range of operating systems from the Haskell home page <http://www.haskell.org>
- We recommend downloading the *Haskell Platform*
- Once installed, the interface GHCi system can be started from the terminal command prompt by simply typing `ghci`.



```
λ > 1+2+3  
6
```

```
λ > 1+2*3  
7
```

```
λ > (1+2)*3  
9
```

```
λ > 2-3+4  
3
```

```
λ > 2*3/4  
1.5
```

```
λ> 2*pi
```

```
6.283185307179586
```

```
λ> (1 + sqrt 5) / 2
```

```
1.618033988749895
```

```
λ> log 2
```

```
0.6931471805599453
```

```
λ> 2^3^4
2417851639229258349412352
```

```
λ> (2^3)^4
4096
```

```
λ> ceiling 2.6
3
```

```
λ> floor 2.6
2
```

```
λ> round 2.6
3
```

```
λ> (sin pi)^2 + (cos pi)^2
1.0
```

```
λ > x = 42
```

```
λ > x+1
```

```
43
```

```
λ > let x = 42 in x+1
```

```
43
```

```
λ > let x = 1 in let x = 2 in x
```

```
2
```

```
λ > x = 1
```

```
λ > x = x+1
```

```
λ > x
```

```
^CInterrupted.
```

```
λ > y = y+1
```

```
λ > y
```

```
^CInterrupted.
```

```
λ> "Haskell rocks!"  
"Haskell rocks!"
```

```
λ> "Haskell " ++ "rocks!"  
"Haskell rocks!"
```

```
λ> "Haskell " <> "rocks!"  
"Haskell rocks!"
```

```
λ> ['H','a','s','k','e','l','l',' ','r','o','c','k','s','!']  
"Haskell rocks!"
```

Command	Meaning
<code>:load <i>name</i></code>	load script <i>name</i>
<code>:reload</code>	reload current script
<code>:set editor <i>name</i></code>	set editor to <i>name</i>
<code>:edit <i>name</i></code>	edit script <i>name</i>
<code>:edit</code>	edit current script
<code>:type <i>expr</i></code>	show type of <i>expr</i>
<code>:?</code>	show all commands
<code>:quit</code>	quit GHCi
...	

```
λ > :type 1  
1 :: Num a => a
```

```
λ > :type 2.5  
2.5 :: Fractional a => a
```

```
λ > :type 5/2  
5/2 :: Fractional a => a
```

```
λ > :type 5 `div` 2  
5 `div` 2 :: Integral a => a
```

```
λ > :type 1+2
```

```
1+2 :: Num a => a
```

```
λ > :type (+)
```

```
(+) :: Num a => a -> a -> a
```

```
λ > :type (1 +)
```

```
(1 +) :: Num a => a -> a
```

```
λ > :type (+ 1)
```

```
(+ 1) :: Num a => a -> a
```



```
λ> :type 2.5
2.5 :: Fractional a => a
```

```
λ> :type 5/2
5/2 :: Fractional a => a
```

```
λ> :type (/)
(/) :: Fractional a => a -> a -> a
```

```
λ> :type (/ 2)
(/ 2) :: Fractional a => a -> a
```

```
λ> :type pi
pi :: Floating a => a
```

```
λ> :type sqrt 2
sqrt 2 :: Floating a => a
```

```
λ> :type cos
cos :: Floating a => a -> a
```

```
λ> fact n = if n == 0 then 1 else n * fact (n-1)
```

```
λ> :type fact
```

```
fact :: (Eq a, Num a) => a -> a
```

```
λ> fact 5
```

```
120
```

```
λ> fact 0
```

```
1
```

```
λ> fact 5.0
```

```
120.0
```

```
λ> fact 2.5
```

```
^CInterrupted.
```

```
λ> f = fact
```

```
λ> :type fact
```

```
fact :: (Eq a, Num a) => a -> a
```

```
λ> f 5
```

```
120
```

```
λ> f (f 3)
```

```
720
```

```
λ> 'a'
'a'
```

```
λ> :type 'a'
'a' :: Char
```

```
λ> 'abc'
error: Syntax error on 'abc'
```

```
λ> 'a':"bc"
"abc"
```

```
λ> "abc"  
"abc"
```

```
λ> :type "abc"  
"abc" :: String
```

```
λ> "abc" ++ "def"  
"abcdef"
```

```
λ> :type (++)  
(++) :: [a] -> [a] -> [a]
```

# Types and classes

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Functional programming concepts

First steps

Types and classes

# Basic concepts

- In Haskell every expression must have a type.
- A **type** is a collection of related values.
- We use the notation  $v :: T$  to mean that  $v$  is a value in the type  $T$ .

## Example

`True :: Bool`

`False :: Bool`

`not :: Bool -> Bool`

`(&&) :: Bool -> Bool -> Bool`

`(||) :: Bool -> Bool -> Bool`



# Basic types

- **Bool** - Logical values.
- **Char** - Single characters.
- **String** - Strings of characters.
- **Int** - Fixed-precision integers.
- **Integers** - Arbitrary-precision integers.
- **Float** - Single-precision floating-point numbers.
- **Double** - Double-precision floating-point numbers.

# List types

- A **list** is a sequence of elements of the **same type**, with the elements being enclosed in square parentheses and separated by commas.
- We write `[T]` for the type of all lists whose elements have type `T`.
- The number of elements in a list is called its **length**.
- The list `[]` of length zero is called the **empty list**.
- `[]` and  `[[] ]` (and `[[[]]]`, `[[[[]]]]`, ...) are different lists.

# List types

```
λ > :type []  
[] :: [a]
```

```
λ > :type [1,2,3,4,5]  
[1,2,3,4,5] :: Num a => [a]
```

```
λ > :type ['a', 'b', 'c', 'd']  
['a', 'b', 'c', 'd'] :: [Char]
```

```
λ > :type ["ab", "cd", "ef", "gh"]  
["ab", "cd", "ef", "gh"] :: [String]
```

```
λ > :type "ab" == :type "cd"  
error: parse error on input ':'
```

# List types

```
λ> :type [cos, sin]
[cos, sin] :: Floating a => [a -> a]
```

```
λ> :type [1, 'a']
error: No instance for (Num Char) arising from the literal '1'
```

```
λ> :type [[1],[2,3],[4,5,6]]
[[1],[2,3],[4,5,6]] :: Num a => [[a]]
```

```
λ> :type [[[1]],[[2,3],[4,5,6]]]
[[[1]],[[2,3],[4,5,6]]] :: Num a => [[[a]]]
```

# Tuple types

- A **tuple** is a sequence of components of possibly **different types**, with the components being enclosed in round parentheses and separated by commas.
- We write  $(T_1, T_2, \dots, T_n)$  for the type of all tuples whose  $i$ -th component have type  $T_i$  for any  $1 \leq i \leq n$ .
- The number of elements in a tuple is called its **arity**.
- The tuple  $()$  of arity zero is called the **empty tuple**.
- Tuple of arity one are not permitted.

# Tuple types

```
λ > :type ()  
() :: ()
```

```
λ > :type (1, 'a')  
(1, 'a') :: Num a => (a, Char)
```

```
λ > :type (1,2, 'a', "abc")  
(1,2, 'a', "abc") :: (Num a, Num b) => (a, b, Char, String)
```

```
λ > :type (sqrt, 'a')  
(sqrt, 'a') :: Floating a => (a -> a, Char)
```

```
λ > :type (1, ('a', "cd"))  
(1, ('a', "cd")) :: Num a => (a, (Char, String))
```

# Tuple types

```
λ > :type (1, ('a', "cd"))  
(1, ('a', "cd")) :: Num a => (a, (Char, String))
```

```
λ > :type (1, [cos, sin])  
(1, [cos, sin]) :: (Floating a1, Num a2) => (a2, [a1 -> a1])
```

```
λ > :type (1)  
(1) :: Num a => a
```

```
λ > let t = (1,2) in (t, 3)  
((1,2),3)
```

```
λ > let t = (1,t)  
error: Couldn't match expected type 'b' with actual type '(a, b)'
```

# Function types

- A **function** is a mapping of one type to results of another type.
- We write  $T1 \rightarrow T2$  for the type of all functions that map arguments of type  $T1$  to results of type  $T2$ .
- There is no restriction that function must be **total** on their argument type.



# Function types

```
λ > :type not
```

```
not :: Bool -> Bool
```

```
λ > :type even -- :type odd
```

```
even :: Integral a => a -> Bool
```

```
λ > :type mod
```

```
mod :: Integral a => a -> a -> a
```

```
λ > add x y = x+y
```

```
λ > :type add
```

```
add :: Num a => a -> a -> a
```

```
λ > add' (x,y) = x+y
```

```
λ > :type add'
```

```
add' :: Num a => (a, a) -> a
```

# Curried functions

- **Currying** is the process of transforming a function that takes multiple arguments in a tuple as its argument, into a function that takes just a single argument and returns another function which accepts further arguments, one by one, that the original function would receive in the rest of that tuple.
- The **function arrow**  $\rightarrow$  in type is assumed to associate to the right.

The type

$T_1 \rightarrow T_2 \rightarrow T_3 \rightarrow \dots \rightarrow T_n$

means

$T_1 \rightarrow (T_2 \rightarrow (T_3 \rightarrow (\dots \rightarrow T_n) \dots))$

# Curried functions

The type

$a1 \rightarrow a2 \rightarrow a3$

means

$a1 \rightarrow (a2 \rightarrow a3)$

# Curried functions

The type

$a1 \rightarrow a2 \rightarrow a3 \rightarrow a4$

means

$a1 \rightarrow (a2 \rightarrow (a3 \rightarrow a4))$

# Curried functions

The type

`a1 -> a2 -> a3 -> a4 -> a5`

means

`a1 -> (a2 -> (a3 -> (a4 -> a5)))`

## Curried functions

Multiplying three integers

```
-- mult :: Int -> (Int -> (Int -> Int))
```

```
mult :: Int -> Int -> Int -> Int
```

```
mult x y z = x*y*z
```

# Curried functions

Multiplying three integers

```
-- mult :: Int -> (Int -> (Int -> Int))
```

```
mult :: Int -> Int -> Int -> Int
```

```
mult x y z = x*y*z
```

```
λ> mult 2 3 4
```

```
24
```

```
λ> :type mult 2
```

```
mult 2 :: Int -> Int -> Int
```

```
λ> :type mult 2 3
```

```
mult 2 3 :: Int -> Int
```

```
λ> :type mult 2 3 4
```

```
mult 2 3 4 :: Int
```

## Curried functions

Multiplying three integers

```
-- mult :: Int -> (Int -> (Int -> Int))
```

```
mult :: Int -> Int -> Int -> Int
```

```
mult x y z = x*y*z
```

```
λ> mult2 = mult 2
```

```
λ> mult3 = mult2 3
```

```
λ> mult3 4
```

```
24
```

```
λ> :type mult2
```

```
mult2 :: Int -> Int -> Int
```

```
λ> :type mult3
```

```
mult3 :: Int -> Int
```



## Polymorphic types

- **Parametric polymorphism** refers to when the type of a value contains one or more (unconstrained) type variables, so that the value may adopt any type that results from substituting those variables with concrete types.
- For example, the function `id :: a -> a` contains an unconstrained type variable `a` in its type, and so can be used in a context requiring `Char -> Char` or `Integer -> Integer` or `(Bool -> Bool) -> (Bool -> Bool)` or any of a literally infinite list of other possibilities.
- The empty list `[] :: [a]` belongs to every list type.

# Polymorphic types

```
λ > length []
```

```
0
```

```
λ > length [1,3,5,7,2,4,6,8]
```

```
8
```

```
λ > length ["Huey", "Dewey", "Louie"]
```

```
3
```

```
λ > length [sin, cos, tan]
```

```
3
```

# Polymorphic types

```
λ> :type length
length :: Foldable t => t a -> Int

λ> :info length
type Foldable :: (* -> *) -> Constraint
class Foldable t where
  length :: t a -> Int
  ...
-- Defined in 'Data.Foldable'
```

# Overloaded types

- A type that contains one or more **class constraints** is called **overloaded**.
- Class constraints are written in the form **C** **a**, where **C** is the name of the class and **a** is a type variable.

# Overloaded types

```
λ > 1 + 2
```

```
3
```

```
λ > :type 1
```

```
1 :: Num a => a
```

```
λ > :type 1 + 2
```

```
1 + 2 :: Num a => a
```

```
λ > 1.0 + 2.0
```

```
3.0
```

```
λ > :type 1.0
```

```
1.0 :: Fractional a => a
```

```
λ > :type 1.0 + 2.0
```

```
1.0 + 2.0 :: Fractional a => a
```

```
λ > sqrt 2 + sqrt 3
```

```
3.1462643699419726
```

```
λ > :type sqrt 2
```

```
sqrt 2 :: Floating a => a
```

```
λ > :type sqrt 2 + sqrt 3
```

```
sqrt 2 + sqrt 3 :: Floating a => a
```

# Overloaded types

```
λ > :type (+)
(+) :: Num a => a -> a -> a
```

```
λ > :type (-)
(-) :: Num a => a -> a -> a
```

```
λ > :type (*)
(*) :: Num a => a -> a -> a
```

```
λ > :type (/)
(/) :: Fractional a => a -> a -> a
```

```
λ > :type sqrt
sqrt :: Floating a => a -> a
```

- A **class** is collection of types that support certain overloaded operations called **methods**.
- Haskell provides a number of basic classes that are built-in to the language.

## Eq – Equality types

This class contains types whose values can be compared for **equality** and **inequality** using the following two methods:

```
(==) :: a -> a -> Bool
```

```
(/=) :: a -> a -> Bool
```

All the basic types **Bool**, **Char**, **String**, **Int**, **Integers**, **Float** and **Double** are instances of the **Eq** class.



## Eq – Equality types

```
λ > True == True  
True
```

```
λ > 'a' == 'b'  
False
```

```
λ > "abc" == "abc"  
True
```

```
λ > 2.5 == 5.2  
False
```

## Eq – Equality types

```
λ> ('a', 1) == ('b', 1)
```

```
False
```

```
λ> (1, 2, 3) == (1, 2)
```

```
error: Couldn't match expected type: (a0, b0, c0) with actual  
      type: (a1, b1)
```

```
λ> [1,2,3] == [1,2,3,4]
```

```
False
```

```
λ> cos == cos
```

```
error: No instance for (Eq (Double -> Double)) arising from a  
      use of '=='
```

## Ord – Ordered types

This class contains types that are instances of the equality class `Eq`, but in addition these values are totally ordered, and as such can be compared using the following six methods:

```
(<)    :: a -> a -> Bool
```

```
(<=)   :: a -> a -> Bool
```

```
(>)    :: a -> a -> Bool
```

```
(>=)   :: a -> a -> Bool
```

```
min    :: a -> a -> a
```

```
max    :: a -> a -> a
```

All the basic types `Bool`, `Char`, `String`, `Int`, `Integers`, `Float` and `Double` are instances of the `Ord` class.

## Ord – Ordered types

```
λ > False < True  
True
```

```
λ > "elegant" < "elephant"  
True
```

```
λ > "a" < "ab"  
True
```

```
λ > 'b' > 'a'  
True
```

```
λ > [1,2,3] <= [1,2]  
False
```

```
λ > [] < [1]  
True
```

# Basic classes

## Ord – Ordered types

```
λ > (1,2) < (1,3)
```

```
True
```

```
λ > (1,2,3) < (1,1)
```

```
error: Couldn't match expected type: (a0, b0, c0) with actual  
      type: (a1, b1)
```

```
λ > [True] < [False,False]
```

```
False
```

```
λ > (False,False) <= (False,True)
```

```
True
```

## Ord – Ordered types

```
λ >
```

```
λ > min ('a',2) ('a',1)  
('a',1)
```

```
λ > max ('a',2) ('a',1)  
('a',2)
```

```
λ > sin < cos
```

```
error: No instance for (Ord (Double -> Double)) arising from a  
      use of '<'
```

```
λ > (1, sin) > (2, cos)
```

```
error: No instance for (Ord (Double -> Double)) arising from a  
      use of '>'
```

## Show – Showable types

This class contains types that can be converted into strings of characters using the following method:

```
show :: a -> String
```

All the basic types `Bool`, `Char`, `String`, `Int`, `Integers`, `Float` and `Double` are instances of the `Show` class.

## Show – Showable types

```
λ > show True  
"True"
```

```
λ > show 'a'  
"'a'"
```

```
λ > show "abc"  
 "\"abc\""
```

```
λ > show [1,2,3]  
"[1,2,3]"
```

```
λ > show (1, True, [1,2,3])  
"(1,True,[1,2,3])"
```



### **Read – Readable types**

This class is dual to **Read** and contains types whose values can be converted from string of characters using the following method:

```
read :: String -> a
```

All the basic types **Bool**, **Char**, **String**, **Int**, **Integers**, **Float** and **Double** are instances of the **Read** class.

### Read – Readable types

```
λ> read "False" :: Bool
False
```

```
λ> read "'a'" :: Char
'a'
```

```
λ> read "\"abc\"" :: String
"abc"
```

```
λ> read "[1,2,3]" :: [Int]
[1,2,3]
```

```
λ> read "(1, True, [1,2,3])" :: (Int, Bool, [Int])
(1,True,[1,2,3])
```

## Num – Numeric types

This class contains types whose values are numeric, and as such can be processed using the following six methods:

(+) :: a -> a -> a

(-) :: a -> a -> a

(\*) :: a -> a -> a

negate :: a -> a

abs :: a -> a

signum :: a -> a

Note that the **Num** class does not provide a division method.

## Num – Numeric types

```
λ > 1+2
```

```
3
```

```
λ > 1-2
```

```
-1
```

```
λ > 1.0+2.0
```

```
3.0
```

```
λ > 2*3
```

```
6
```

```
λ > 2.0*3.0
```

```
6.0
```

### Num – Numeric types

```
λ > negate 3.0  
-3.0
```

```
λ > negate (-2)  
2
```

```
λ > abs(-1.5)  
1.5
```

```
λ > signum 3  
1
```

```
λ > signum (-3)  
-1
```

## Integral – Integral types

This class contains types that are instances of the numeric class `Num`, but in addition whose values are integers, and as such support the method of integer division and integer remainder:

```
div :: a -> a -> a
```

```
mod :: a -> a -> a
```

## Integral – Integral types

```
λ > div 7 2
```

```
3
```

```
λ > 7 `div` 2
```

```
3
```

```
λ > 8 `div` 2
```

```
4
```

```
λ > 7 `mod` 2
```

```
1
```

```
λ > 8 `mod` 2
```

```
0
```

### Integral – Integral types

```
λ > (-7) `div` 2  
-4
```

```
λ > (-7) `div` (-2)  
3
```

```
λ > (-7) `mod` 2  
1
```

```
λ > (-7) `mod` (-2)  
-1
```



## Fractional – Fractional types

This class contains types that are instances of the numeric class **Num**, but in addition whose values are non-integral, and as such support the method of integer fractional division and fractional reciprocation:

```
(/) :: a -> a -> a
```

```
recip :: a -> a -> a
```

The basic types **Float** and **Double** are instances of the **Fractional** class.

## Fractional – Fractional types

```
λ> 7.0 / 2.0  
3.5
```

```
λ> 2.0 / 7.0  
0.2857142857142857
```

```
λ> recip 2.0  
0.5
```

```
λ> recip 1.0  
1.0
```